A Fresh Look at Equivalent Fractions

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Hello Teachers,

Rational numbers are challenging for students to understand. For one thing, there is a never-ending supply of equivalent fractions that share a place on the number line. Understanding that equivalent fractions represent the same number is one of the conceptual puzzles students must piece together in order to work with fractions.

Arranged in a logical progression that gradually increases in complexity, **A Fresh Look at Equivalent Fractions** invites students to use familiar skills and concepts in order to make sense of fractions. The well known format of a multiplication chart mixed with basic mental math reveals patterns underlying equivalent fractions.

A Fresh Look at Equivalent Fractions focuses students' attention on a key math rule that makes it possible for so many seemingly different fractions to represent the same number. I hope you and your students find this fresh look at equivalent fractions valuable and worthwhile.

If you have questions or comments about these activities, please email me at Isabelle@UnCommon-Core.com. If you like this book, please leave a wildly enthusiastic review on my Teachers Pay Teachers store.

Thank you,

Isabelle

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A Fresh Look at Equivalent Fractions

A Fresh Look at Equivalent Fractions takes some of the mystery out of equivalent fractions by inviting students to investigate how equivalent fractions are made before trying to reduce them to their simplest form, or prove their equivalence.

The *multiplicative identity property of one* says that a number's identity does not change when that number is multiplied by one. Starting with the simplest form and then multiplying by one to create a sequence of equivalent fractions, students complete a series of multiplication charts in which each row is filled with equivalent fractions. They also check numerators and denominators for common factors, numerical relationships, or other clues to what the fraction will look like when reduced to simplest form.

When students create a sequence of equivalent fractions and then see those equivalent fractions side by side, they gain insight into how equivalent fractions are generated, why they represent the same number, and why they share the same place on a number line.

Equivalent fractions:

- reduce to the same simplest form,
- have the same quotient when divided,
- occupy the same place on the number line,
- represent the same proportion,
- share a product when cross multiplied,
- can cause problems for elementary students,
- and their teachers!

People, like fractions, use different titles, forms of address, and names depending on the circumstances.

A teacher might be called 'Ms. Green' by her students, 'mom' by her children, 'honey' by her spouse, 'sis' by a sibling, or 'ma'am' by a cashier.

She may receive mail addressed to:

- Dear Neighbor
- Library Patron
- Pizza Lover
- Registered Voter
- Valued Customer
- New Subscriber
- Car Owner
- Gardening Enthusiast
- Coupon Clipper
- Desk Dweller
- Dear Alum
- Current Resident

Fractions From Various Points of View

There are many ways to represent the same number. Rational numbers can be written as fractions, division problems, statements in English, or as the quotient when the numerator is divided by the denominator. They can be shown as a place on the number line, pieces of a whole, or parts of a set.

In order to work with equivalent fractions, students must first be able to identify rational numbers from various points of view, connecting each different representation of the number with the others.

Proper Fractions: Various Points of View

Show a proper fraction from different points of view.

Example

Fraction

$\frac{1}{2}$

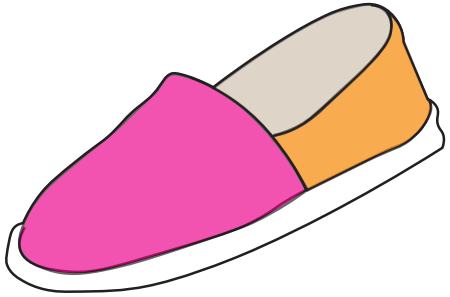
one-half

Simplest Form

$\frac{1}{2}$

Decimal

.5



one shoe

English

half of a pair of shoes

Area

Proper Fractions: Various Points of View

Teacher Tips

Fraction

You could write a fraction for your students to work with before you copy the page. Alternatively, you might let each student write their own.

Make sure your students see the numerator as the number of individual items and the denominator as the number needed to make one entire group, pair, trio, or set.

Simplest Form

Start with a fraction that is written in simplest form. Once your students show the various points of view for that number, then you might decide to give them a more challenging number next time.

Decimal

If your students are not ready to find decimals, use white out to 'erase' this term before you copy the worksheet.

Use the center space for making pictures of the fraction.

Encourage students to find creative ways to picture the fraction.

Ask them to explain how different ways to express the number are related.

English

Invite students to record all of the everyday and academic language that can be used to describe the number.

Let your students use the back of the page to show fractions on a number line.

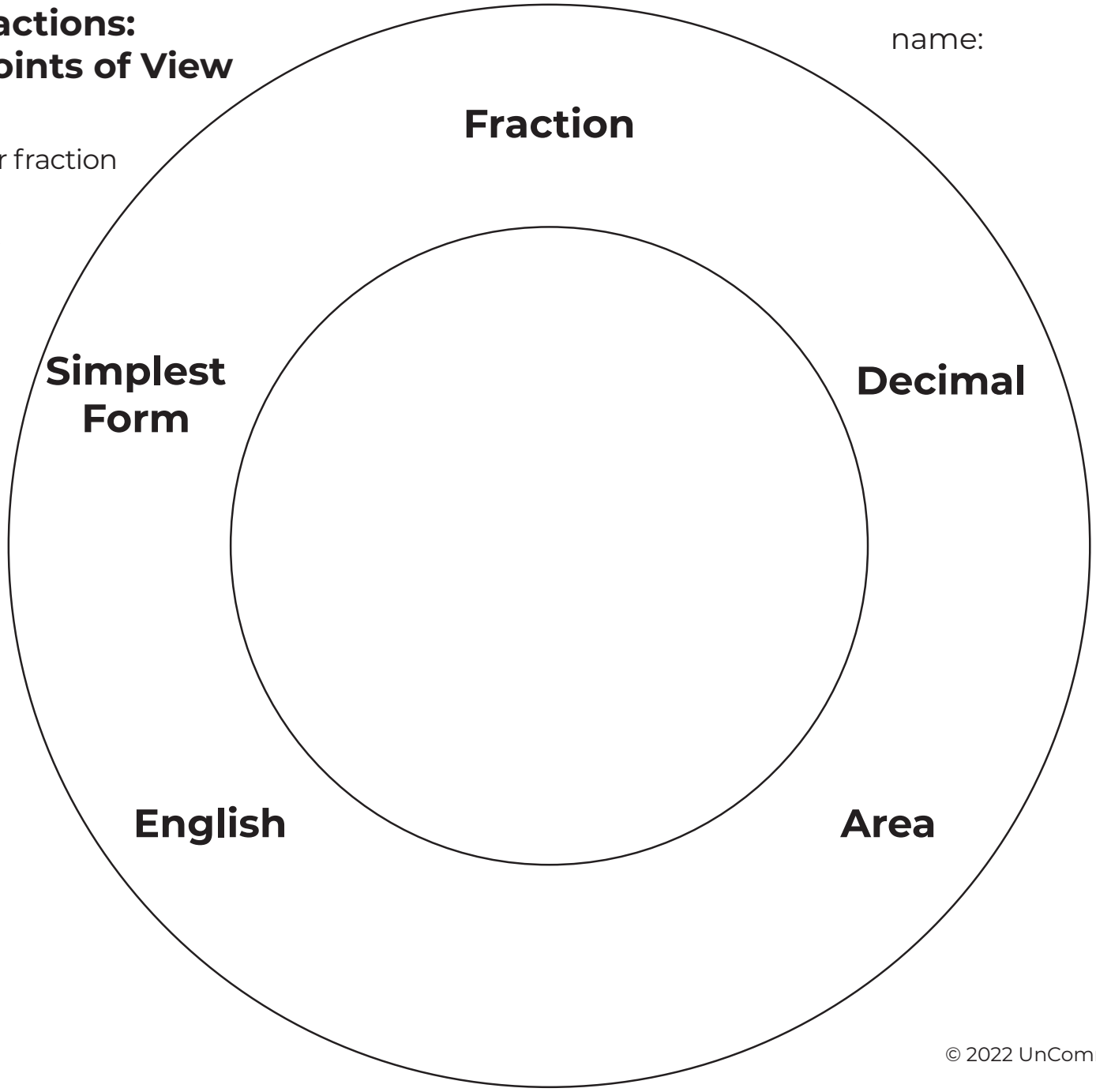
Area

Have your students create a model to show the area of the fraction. Circles would be the easiest place to start.

Proper Fractions:
Various Points of View

name:

Show a proper fraction from different points of view.



Fractions Equivalent to One

Fractions with the same numerator and denominator equal one. Students must grasp this idea in order to use the *multiplicative identity property of one* to make sense of equivalent fractions.

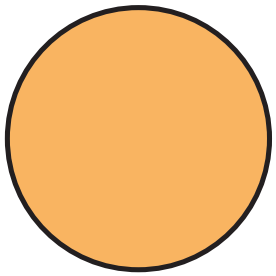
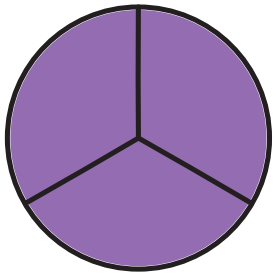
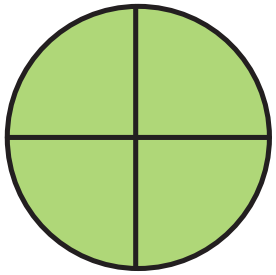
Plan extra time for your students to investigate this relationship between numerators and denominators before moving on. Use the time to help your students solidify their use of academic vocabulary, fractional notation, and placing fractions on the number line.

Once students understand this concept they will enjoy playing around with larger and larger numbers. Recognizing that seven million over seven million is equivalent to one gives young students a sense of pride in their ability to wrangle large numbers.

Which Fractions are Equivalent to One?

EXAMPLE

Equivalent fractions are the same number.
They have the same area, the same place on the number line, the same product when cross multiplied,and the same quotient when the numerator is divided by the denominator.

one		 $1 \div 1 = 1$	$\frac{1}{1}$
$\frac{3}{3}$		 $3 \div 3 = 1$	$\frac{1}{1} \quad \frac{3}{3} \quad \frac{3}{3}$
$\frac{4}{4}$		 $4 \div 4 = 1$	$\frac{1}{1} \quad \frac{4}{4} \quad \frac{4}{4}$

Which Fractions are Equivalent to One?

Teacher Tips

Before you make copies, write in some fractions equivalent to one for your students to verify. Keep a blank copy-master handy so you can reuse this worksheet several times. To make it more difficult, include fractions that are and are not equal to one.

As always, you could give your class a blank copy of this worksheet and have them fill in the fractions they would like to test. They could make up some fractions by tossing a pair of dice or by looking around for some numbers on display in the classroom.

After having used this worksheet in class, you might decide to create another version of it for homework. Additionally, you could reuse it later in the year for review or even as an assessment. Being able to identify fractions equivalent to one is an essential skill.

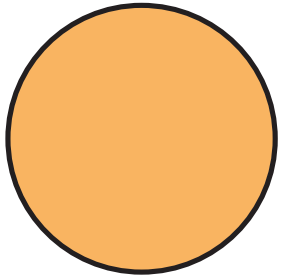
Can you identify a fraction that is equivalent to one by looking at the numerator and denominaor? Will this method work every time? Why do you think so?

In order to benefit from **A Fresh Look at Equivalent Fractions,** your students must be convinced that any fraction in which the numerator and the denominator are the same number is equal to one.

Which Fractions are Equivalent to One?

name:

Equivalent fractions are the same number.
They have the same area, the same place on the number line, the same product when cross multiplied,and the same quotient when the numerator is divided by the denominator.

one	  $1 \div 1 = 1$ $\frac{1}{1}$

Which Fractions are Equivalent to One?

Can you identify a fraction that is equivalent to one by looking at the numerator and denominaor? Will this method work every time? Why do you think so?

Unit Fractions

Fractions that have a one in the numerator are an excellent starting point for studying equivalent fractions. Unit fractions provide an uncomplicated view of what happens when numbers are multiplied or divided by one. This simple example of the *multiplicative identity property of one* lets students develop a basic understanding of why numbers stay the same when multiplied or divided by one.

Students can easily identify a unit fraction in its simplest form. As they multiply the unit fraction by the sequence of fractions equivalent to one students can see the plus one pattern in the numerators. The skip counting patterns in the denominators become clear with only a little bit of effort.

Multiply to Find Equivalents for Unit Fractions: Example

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	$\frac{7}{21}$	$\frac{8}{24}$	$\frac{9}{27}$	$\frac{10}{30}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$	$\frac{6}{24}$	$\frac{7}{28}$	$\frac{8}{32}$	$\frac{9}{36}$	$\frac{10}{40}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{10}$	$\frac{3}{15}$	$\frac{4}{20}$	$\frac{5}{25}$	$\frac{6}{30}$	$\frac{7}{35}$	$\frac{8}{40}$	$\frac{9}{45}$	$\frac{10}{50}$

Multiply to Find Equivalents for Unit Fractions:

Teacher Tips

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$										
$\frac{1}{2}$										
$\frac{1}{3}$										
$\frac{1}{4}$										
$\frac{1}{5}$										

Let your students choose a couple fractions from each row and then demonstrate that the fractions are equivalent. Have them practice cross multiplication to show that the fractions are the same number.

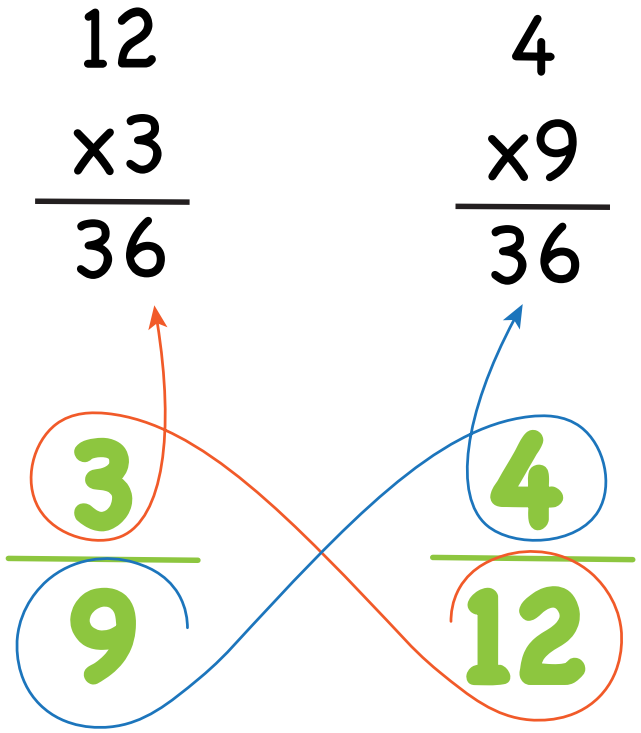
Fill the lefthand column with unit fractions you would like your students to multiply by one.

Ask your students to explain why the numbers in each column have the same numerator.

Challenge your students to use what they learned by completing this chart to write out step by step instructions for finding the simplest form of a fraction. Start with a concrete example.

Invite your students to look at the fractions in the first and last columns. How is multiplying by ten over ten different than and similar to multiplying by one over one? What will happen when multiplying by 100 over 100?

Let your students choose a couple fractions from each row and then demonstrate that the fractions are equivalent. Have them practice cross multiplication to show that the fractions are the same number.



Multiplying the denominators of one fraction by the numerators of another indicates the relative size of the fractions. If the products are equal then the two original fractions are the same number.

Ask your students to explain why the numbers in each column have the same numerator.

Fractions equivalent to one are located in the top cell of each column.

$\frac{9}{9}$

When these fractions are multiplied by a unit fraction, the numerators of the products are all the same, too.

$\frac{9}{9}$

$\frac{9}{18}$

They have to be the same because all unit fractions have a one in the numerator.

$\frac{9}{27}$

$\frac{9}{36}$

$\frac{9}{45}$

Invite your students to look at the fractions in the first and last columns. How is multiplying by ten over ten different than and similar to multiplying by one over one? What will happen when multiplying by 100 over 100?

$$\frac{1}{2} \quad \frac{10}{20} \quad \frac{100}{200} \quad \frac{1000}{2000}$$

$$\frac{1}{3} \quad \frac{10}{30} \quad \frac{100}{300} \quad \frac{1000}{3000}$$

$$\frac{1}{4} \quad \frac{10}{40} \quad \frac{100}{400} \quad \frac{1000}{4000}$$

$$\frac{1}{5} \quad \frac{10}{50} \quad \frac{100}{500} \quad \frac{1000}{5000}$$

Help your students recall what they know about place value and multiplying by numbers that end in zero.

Guide them to see how the information they learned about place value can be useful when thinking about fractions.

Challenge your students to use what they learned by completing this chart to write out step by step instructions for finding the simplest form of a fraction. Start with a concrete example.

The product of one-third times four-fourths is four-twelfths. One-third is equivalent to four-twelfths and it is the simplest form of four-twelfths.

Working backwards, starting with four-twelfths, how could we find its simplest form?

What numbers divide four and twelve evenly? In other words, do four and twelve have any common factors?

Four is a factor of itself and of twelve. Dividing both four and twelve by four does not change the number because four-fourths is equivalent to one.

$$\frac{4}{12} \div \frac{4}{4} = \frac{1}{3}$$

Multiply to Find Equivalents for Unit Fractions:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$										
$\frac{1}{2}$										
$\frac{1}{3}$										
$\frac{1}{4}$										
$\frac{1}{5}$										

Multiply to Find Equivalents for Unit Fractions: name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

Fractions Representing Whole Numbers

Just as having a one in the numerator makes working with equivalents to unit fractions easier, having a one in the denominator can do the same thing. The presentation of equivalent fractions is less complex due to having a one in the simplest form of the fraction.

Working with fractions that represent whole numbers gives students another basic example of the *multiplicative identity property of one* in action. Students can easily identify a whole number in its simplest fractional form. As they multiply the whole number fractions by various equivalents to one, students can see the plus one pattern develop in the denominators. The skip counting pattern in the numerators becomes clear with only a little bit of effort.

Multiply to Find Fractions Equivalent to Whole Numbers:

Example

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{2}{1}$	$\frac{2}{1}$	$\frac{4}{2}$	$\frac{6}{3}$	$\frac{8}{4}$	$\frac{10}{5}$	$\frac{12}{6}$	$\frac{14}{7}$	$\frac{16}{8}$	$\frac{18}{9}$	$\frac{20}{10}$
$\frac{3}{1}$	$\frac{3}{1}$	$\frac{6}{2}$	$\frac{9}{3}$	$\frac{12}{4}$	$\frac{15}{5}$	$\frac{18}{6}$	$\frac{21}{7}$	$\frac{24}{8}$	$\frac{27}{9}$	$\frac{30}{10}$
$\frac{4}{1}$	$\frac{4}{1}$	$\frac{8}{2}$	$\frac{12}{3}$	$\frac{16}{4}$	$\frac{20}{5}$	$\frac{24}{6}$	$\frac{28}{7}$	$\frac{32}{8}$	$\frac{36}{9}$	$\frac{40}{10}$
$\frac{5}{1}$	$\frac{5}{1}$	$\frac{10}{2}$	$\frac{15}{3}$	$\frac{20}{4}$	$\frac{25}{5}$	$\frac{30}{6}$	$\frac{35}{7}$	$\frac{40}{8}$	$\frac{45}{9}$	$\frac{50}{10}$

Multiply to Find Fractions Equivalent to Whole Numbers:

Teacher Tips

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$										
$\frac{2}{1}$										
$\frac{3}{1}$										
$\frac{4}{1}$										
$\frac{5}{1}$										

The numbers in each row are equivalent to each other. The first number in the row shows the simplest form. Multiplying a number by one does not change that number.

How can students tell that these fractions are equal to whole numbers? Why are they the only improper fractions that do not reduce to a mixed number?

Ask your students why the numbers in each column have the same denominators.

Invite your students to reflect on how knowing the multiplication tables can help them reduce equivalent fractions to their simplest form. Invite them to start with some of the equivalent fractions they found by multiplying and work backwards to find the simplest form.

Invite your students to explain why the numbers in each row are equivalent to each other and why the first number in the row shows the simplest form.

Fractions equivalent to one are located in the top cell of each column. Make sure that your students understand why a number divided by itself is equal to one.

The fraction farthest to the left in each column is the simplest form because the numerator and denominator are the smallest combination of integers that express that proportion.

Every fraction in each row has to be equivalent because they were created by multiplying the fraction in simplest form by various equivalents to one.

How can students tell that these fractions are equal to whole numbers? Why are they the only improper fractions that do not reduce to a mixed number?

Using their knowledge of the multiplication table, students can see that the numerator is a multiple of the denominator.

Since the denominator will divide into the the numerator without leaving a remainder, these improper fractions can be reduced to whole numbers and not to mixed numbers.

Ask your students why the numbers in each column have the same denominators.

Fractions equivalent to one are located in the top cell of each column.

When these fractions are multiplied by a fraction equal to a whole number in simplest form, the denominators in each column will all be the same.

In simplest form, fractions equal to whole numbers have a one in the denominator.

Invite your students to reflect on how knowing the multiplication tables can help them reduce equivalent fractions to their simplest form. Invite them to start with some of the equivalent fractions they found by multiplying and work backwards to find the simplest form.

Knowing the multiplication table, allows students to identify numerical relationships between numerators and denominators, find improper fractions that are equivalent to whole numbers, or to figure out when the numerator and denominator have factors in common.

Multiply to Find Fractions Equivalent to Whole Numbers:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{1}{1}$										
$\frac{2}{1}$										
$\frac{3}{1}$										
$\frac{4}{1}$										
$\frac{5}{1}$										

Multiply to Find Fractions Equivalent to Whole Numbers:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

Proper Fractions

Everything students learned from multiplying unit fractions and whole numbers by one, can also be applied to proper fractions.

They have seen the *multiplicative identity property of one* in action and are accustomed to looking for a skip counting pattern in the numerators or denominators of equivalent fractions.

Now students will continue to create equivalent fractions using the same process as before. This time they will examine the numerator and denominator of each equivalent fraction for predictable relationships such as; shared factors, identical proportions, or relative size.

Multiply to Find Equivalent Proper Fractions: Example

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$	$\frac{21}{28}$	$\frac{24}{32}$	$\frac{27}{36}$	$\frac{30}{40}$
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{4}{10}$	$\frac{6}{15}$	$\frac{8}{20}$	$\frac{10}{25}$	$\frac{12}{30}$	$\frac{14}{35}$	$\frac{16}{40}$	$\frac{18}{45}$	$\frac{20}{50}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{6}{10}$	$\frac{9}{15}$	$\frac{12}{20}$	$\frac{15}{25}$	$\frac{18}{30}$	$\frac{21}{35}$	$\frac{24}{40}$	$\frac{27}{45}$	$\frac{30}{50}$
$\frac{4}{5}$	$\frac{4}{5}$	$\frac{8}{10}$	$\frac{12}{15}$	$\frac{16}{20}$	$\frac{20}{25}$	$\frac{24}{30}$	$\frac{28}{35}$	$\frac{32}{40}$	$\frac{36}{45}$	$\frac{40}{50}$

Multiply to Find Equivalent Proper Fractions:

Teacher Tips

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{2}{3}$										
$\frac{3}{4}$										
$\frac{2}{5}$										
$\frac{3}{5}$										
$\frac{4}{5}$										

Unit fractions are also proper fractions. None are included here in order to give students more practice with other numbers. Your students might benefit from including unit fractions with the other proper fractions.

There are many proper fractions which could be placed in the left hand column. Why not give your students blank pages and let them choose their own?

Have your students complete this chart using mental math while looking for patterns in the numerators and denominators.

Invite your students to make flash cards of the fractions on this chart. Have them practice finding the simplest form for each fraction, grouping equivalent fractions together, or placing some of the fractions in ascending order.

Invite your students to make flash cards of the fractions on this chart. Have them practice finding the simplest form for each fraction. Have them group the equivalent fractions together or choose three or four fractions and put them in ascending order.

To reduce a fraction to its simplest form, divide by fractions equivalent to one that are made up of factors common to both the numerator and denominator. Keep going until the numerator and denominator are as small as possible.

$$\frac{12}{20} \div \frac{2}{2} = \frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

$\frac{3}{5}$ $\frac{9}{15}$ $\frac{12}{20}$

Fractions that reduce to the same simplest form are equivalent to each other - they are the same number.

When fractions have the same numerator, the number with the largest denominator is the smallest fraction.

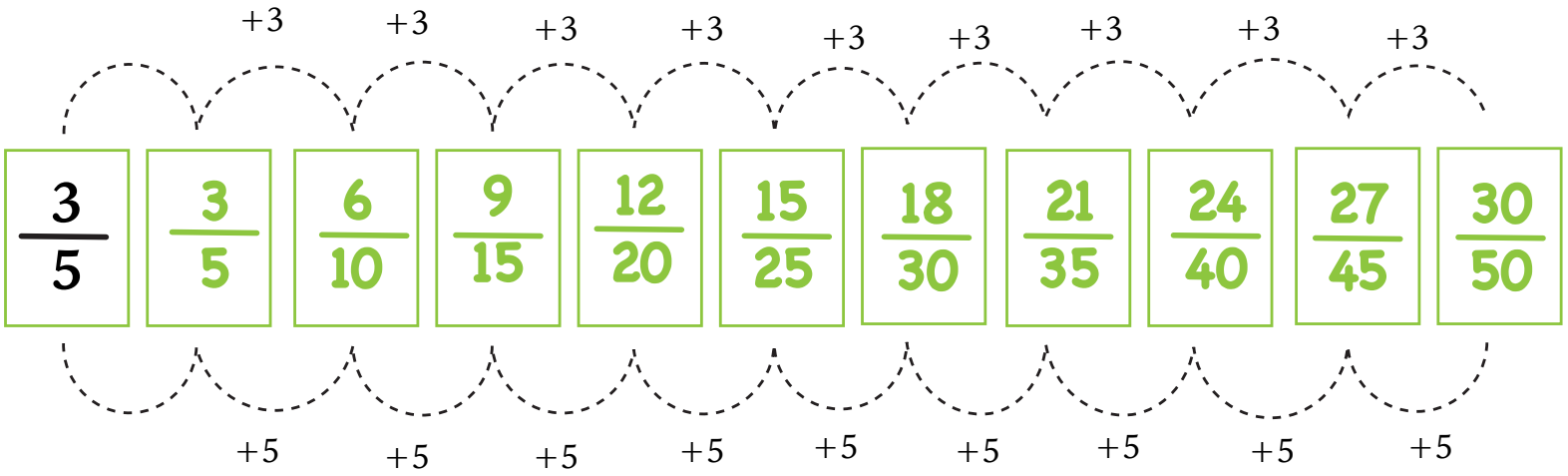
$\frac{6}{15}$ $\frac{6}{10}$ $\frac{6}{9}$ $\frac{6}{8}$

$\frac{6}{15}$ $\frac{9}{15}$ $\frac{12}{15}$

When fractions have the same denominator, the number with the largest numerator is the largest fraction.

Have your students complete this chart using mental math while looking for patterns in the numerators and denominators.

In fractions equivalent to three-fifths, there is a plus three pattern in the numerators. This is created when the three in the numerator is multiplied by fractions equivalent to one that have been arranged sequentially. This is the same as the three's times table.



Similarly, there is a familiar plus five pattern in the denominators which is found when skip counting by five. Multiplying by consecutive versions of fractions equivalent to one naturally creates this pattern.

Multiply to Find Equivalent Proper Fractions: name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{2}{3}$										
$\frac{3}{4}$										
$\frac{2}{5}$										
$\frac{3}{5}$										
$\frac{4}{5}$										

Multiply to Find Equivalent Proper Fractions: name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

Improper Fractions

Following the pattern that has been established in the previous activities, students will use the *multiplicative identity property of one* to find equivalent fractions for improper fractions.

They will practice searching for common factors between numerator and denominator of each equivalent fraction. They will also consider thinking through the process 'backwards' in order to reduce improper fractions to their simplest fractional form.

Multiply to Find Equivalent Improper Fractions:

Example

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{6}{4}$	$\frac{9}{6}$	$\frac{12}{8}$	$\frac{15}{10}$	$\frac{18}{12}$	$\frac{21}{14}$	$\frac{24}{16}$	$\frac{27}{18}$	$\frac{30}{20}$
$\frac{4}{3}$	$\frac{4}{3}$	$\frac{8}{6}$	$\frac{12}{9}$	$\frac{16}{12}$	$\frac{20}{15}$	$\frac{24}{18}$	$\frac{28}{21}$	$\frac{32}{24}$	$\frac{36}{27}$	$\frac{40}{30}$
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{10}{4}$	$\frac{15}{6}$	$\frac{20}{8}$	$\frac{25}{10}$	$\frac{30}{12}$	$\frac{35}{14}$	$\frac{40}{16}$	$\frac{45}{18}$	$\frac{50}{20}$
$\frac{5}{3}$	$\frac{5}{3}$	$\frac{10}{6}$	$\frac{15}{9}$	$\frac{20}{12}$	$\frac{25}{15}$	$\frac{30}{18}$	$\frac{35}{21}$	$\frac{40}{24}$	$\frac{45}{27}$	$\frac{50}{30}$
$\frac{5}{4}$	$\frac{5}{4}$	$\frac{10}{8}$	$\frac{15}{12}$	$\frac{20}{16}$	$\frac{25}{20}$	$\frac{30}{24}$	$\frac{35}{28}$	$\frac{40}{32}$	$\frac{45}{36}$	$\frac{50}{40}$

Multiply to Find Equivalent Improper Fractions:

Teacher Tips

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

Write in any improper fractions for your class to multiply before you make copies.

Challenge your students to think 'backwards' through the process of creating equivalent fractions in order to reduce an equivalent fraction back to its simplest form.

Improper fractions can also be referred to as 'top-heavy' fractions. Ask your students how this alternative name describes improper fractions.

Invite your students to look at the relationship between the numerator and denominator. How can they tell these numbers are larger than one?

Invite your students to explain what they see going on in this multiplication chart. Which patterns did they find that made completing the chart with mental math easier?

Invite your students to explain what they see going on in this multiplication chart. Which patterns did they find that made completing the chart with mental math easier?

Multiplying improper fractions by consecutive versions of numbers equivalent to one creates the same patterns as those found when skip counting or multiplying integers. This is why knowing the multiplication tables makes finding equivalent fractions much easier.

Your students may have noticed other patterns. Please share them on the UnCommon-Core.com Facebook page.

How can your students tell these numbers are larger than one? Why can improper fractions be described as 'top-heavy'?

When the numerator is equal to or larger than the denominator, the number is an improper fraction. Improper fractions will reduce to a whole number or a mixed number. Since the numerator is larger than the denominator, they can be thought of as 'top-heavy.'

Challenge your students to think 'backwards' through the process of creating equivalent fractions in order to reduce an equivalent fraction back to its simplest form.

A Fresh Look at Equivalent Fractions shows students how to generate a series of equivalent fractions by multiplying the simplest form by fractions equal to one.

Since division is the opposite of multiplication, reversing the process involves dividing the number by a fraction equivalent to one. But which fractional equivalent to one should they use?

If students begin with one of the equivalent fractions and divide it by a number equivalent to one that has been made up of the largest factor shared by both the numerator and denominator, they will end up with the simplest form. Students who get into the habit of looking for the greatest common factors between the numerator and denominator will find this process easier. This is another reason why memorising the multiplication tables helps students calculate with fractions.

Multiply to Find Equivalent Improper Fractions:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$
$\frac{3}{2}$										
$\frac{4}{3}$										
$\frac{5}{2}$										
$\frac{5}{3}$										
$\frac{5}{4}$										

Multiply to Find Equivalent Improper Fractions:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

More Fractions from Various Points of View

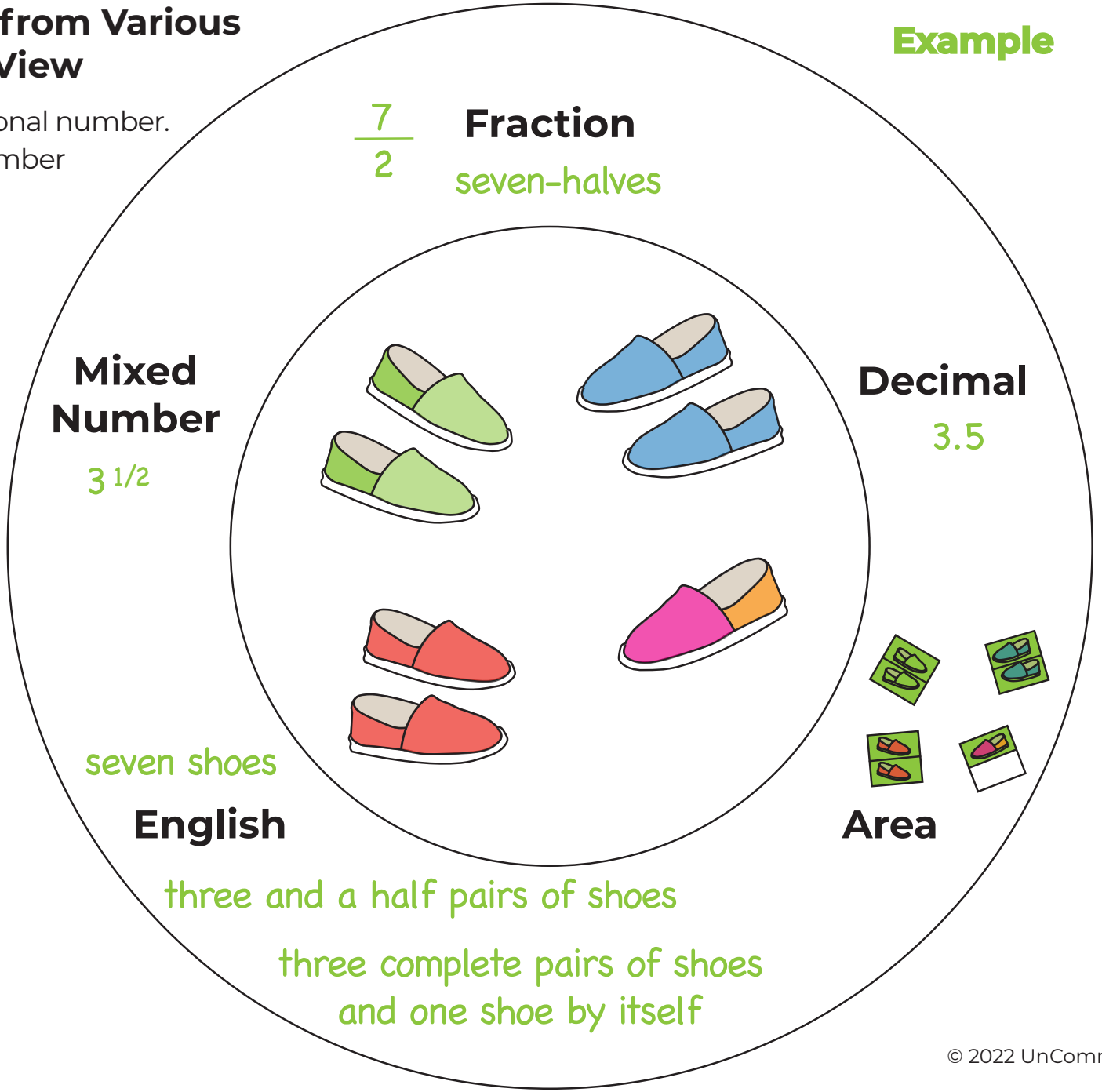
Rational numbers can be written as fractions, decimals, mixed numbers, or ratios. They can be modeled as a place on the number line, parts of a whole, parts of a set, or using plain English.

Students need copious amounts of experience connecting the various representations of fractions together. Before they compare fractions or try to determine which fractions might be equivalent to each other, they need to be adept at recognizing fractional numbers from different points of view.

Fractions from Various Points of View

Think of a rational number. Show that number from different points of view.

Example



Fractions from Various Points of View

Teacher Tips

Have students create an image of their number in the center. They could draw, use stickers, stamp, or find other creative ways to model the number.

To get started, encourage them to think about items that naturally come in sets of certain amounts. Some flowers have five petals, there are eight notes in an octave, many items are sold by the dozen, and so on.

Encourage them to see how each different way to express the number is related.

Fraction

Make sure your students see the numerator as the number of individual items and the denominator as the number needed to make one entire group, pair, trio, or set.

Decimal

Have your students write an explanation of how they found the decimal by dividing the numerator by the denominator.

Area

Have your students create a model to show the area of the fraction. Ask them the difference between modeling the area of proper and improper fractions.

English

Invite students to record all of the everyday and academic language that can be used to describe the number.

Mixed Number

Can your students explain where each part of the mixed number came from? The remainder from the division problem becomes the numerator in the fractional part of the mixed number. The denominators for the original fraction and the fractional part of the mixed number are the same.

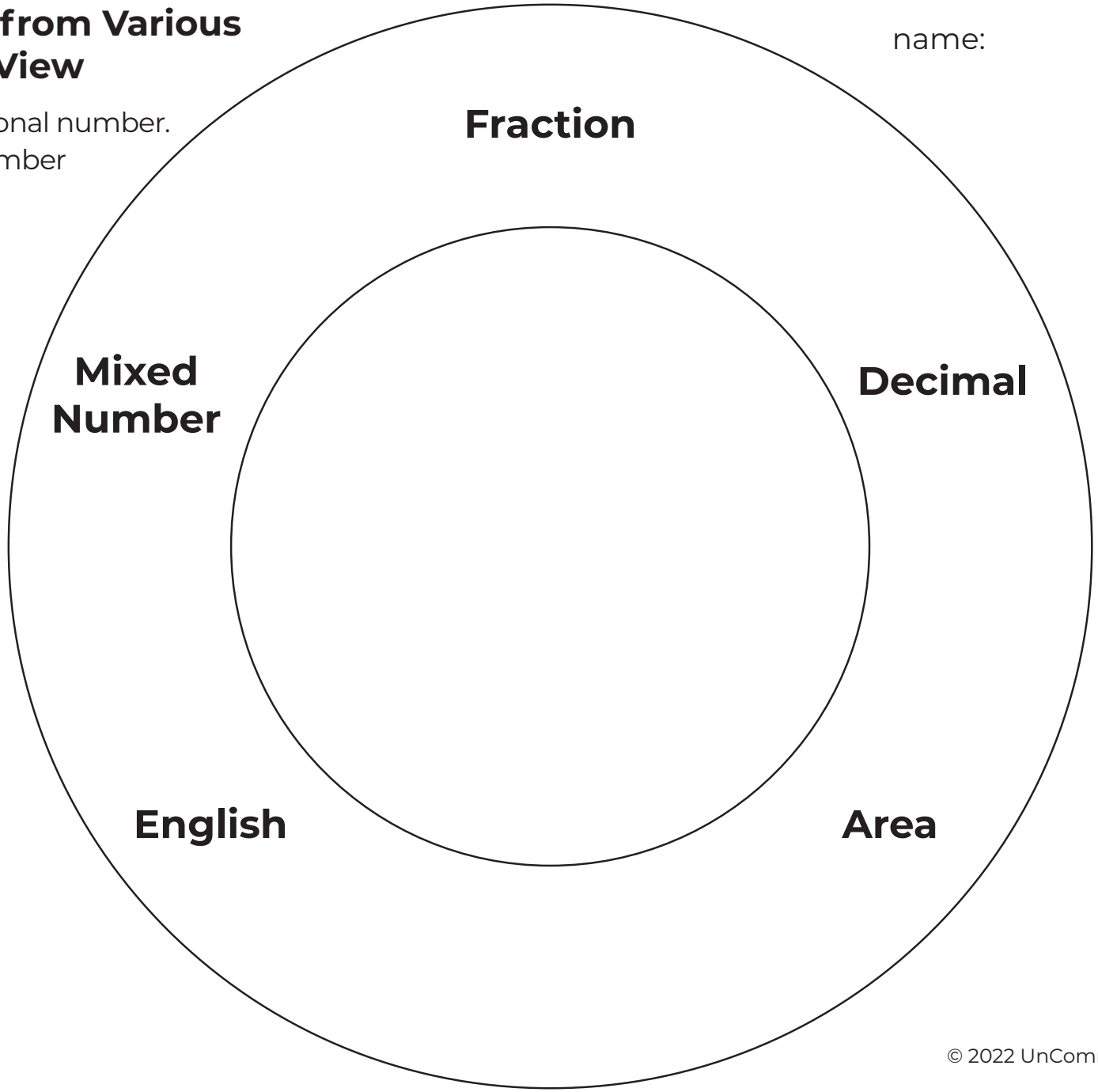
Ask your students why proper fractions cannot be mixed numbers. Which improper fractions cannot be reduced to a mixed number?

Let your students use the back of the page to show fractions on a number line or model fractions.

Fractions from Various Points of View

name:

Think of a rational number. Show that number from different points of view.



Are These Fractions Equivalent?

As soon as students are adept at working with fractions in different forms, they are ready to compare fractions in order to determine which are equivalent.

Equivalent fractions will have the same area, the same quotient when the numerator is divided by the denominator, the same decimal form, the same simplest form, the same place on the number line, and the same ratio. Equivalent fractions represent the same proportion.

Equivalent fractions are the same number.

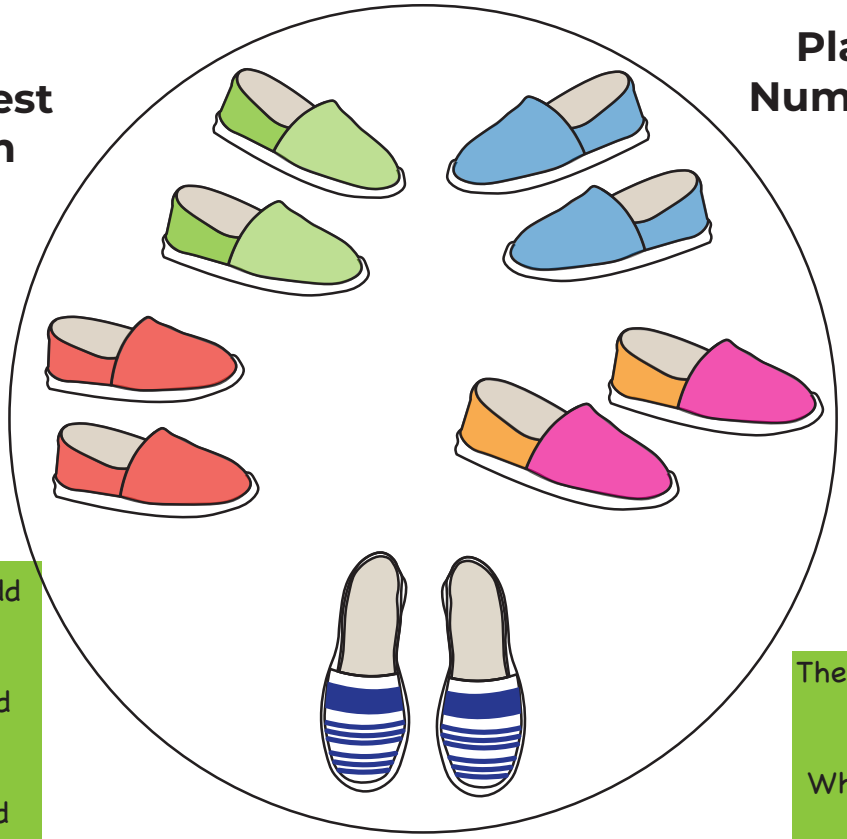
Are These Fractions Equivalent?

Example

Fractions

$\frac{4}{2}$ $\frac{6}{3}$ $\frac{10}{5}$

These three fractions are equivalent.



Simplest Form

The simplest form of all three fractions is 2 over 1. Those are the smallest numerator and denominator that show the proportion in which the numerator is twice as large as the denominator.

Place on Number Line

2

4 hops of one-half each is 2

6 hops of one-third each is 2

10 hops of one-fifth each is 2

Area

Four halves of a square add up to two squares

Six thirds of a square add up to two squares

Ten fifths of a square add up to two whole squares.

Quotient, Percent, Proporton

The quotient when the denominator divides the numerator is 2.

When the denominator is 100, the numerator is 200.

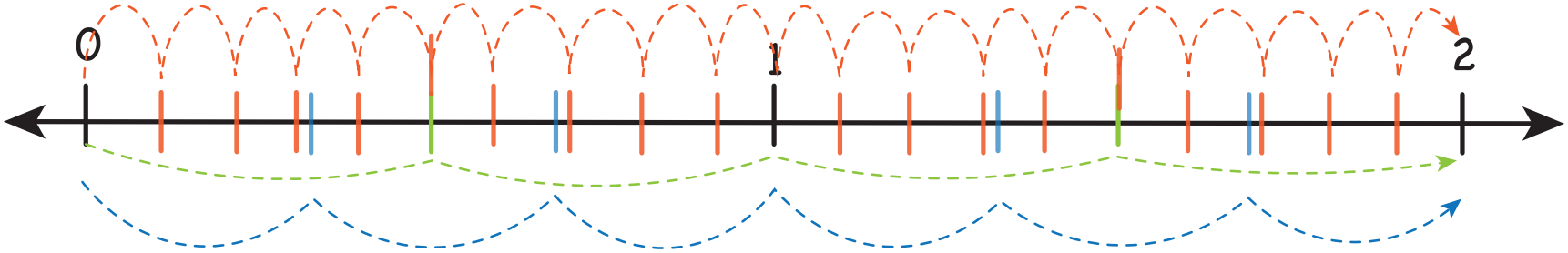
In all fractions, the numerator is twice as large as the denominator.

Product when cross multiplied

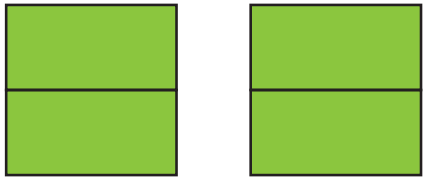
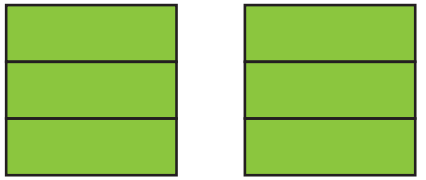
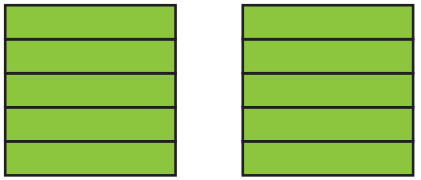
4x3=12 and 2x6=12
6x5=30 and 3x10=30
4x5=20 and 2x10=20

Are These Fractions Equivalent?

Example



These three fractions are equivalent: They have the same place on the number line, the same area, the same proportion, quotient, and percent. They reduce to the same simplest form.

$\frac{4}{2}$	 four shoes/two pairs	$\frac{4}{2} \times \frac{50}{50} = \frac{\quad}{\quad}$ Four is double the amount of two; two is half the amount of four	$2\overline{)4}$	
$\frac{6}{3}$	 six shoes/three pairs	$\frac{6}{3} \times \frac{33.3}{33.3} = \frac{\quad}{\quad}$ the numerator is twice the denominator; three is half of six	$3\overline{)6}$	
$\frac{10}{5}$	 ten shoes/five pairs	$\frac{10}{5} \times \frac{20}{20} = \frac{\quad}{\quad}$ Five times two is ten, ten divided by two is five	$5\overline{)10}$	

Are These Fractions Equivalent?

Think of equivalent fractions. Show different ways to see if they are the same number.

Fraction

Write in the fractions that you want your students to test for equivalence before you make copies.

Teacher Tips

Place on Number Line

Sharing a space on the number line is the mathematical definition of equivalent fractions.

Quotient, Percent, Proportion

You might want to white out all or some of these options before copying the handout for your students.

Product when cross multiplied

Make sure your students are familiar with this method first.

Area

Make sure that your students start with the same size shapes.

Simplest Form

Help your students see why fractions that reduce to the same simplest form are equivalent.

In some cases having an illustration of the fractions or of the area could help students visualize the proportions represented.

This space could also be used to show their math work or explain their answer.

Are These Fractions Equivalent?

Teacher Tips

Use this space to let your students show the location of each fraction on a number line.
By now your students should have lots of experience placing fractions on number lines.
If not, let them practice before asking them to place three fractions on the same number line.

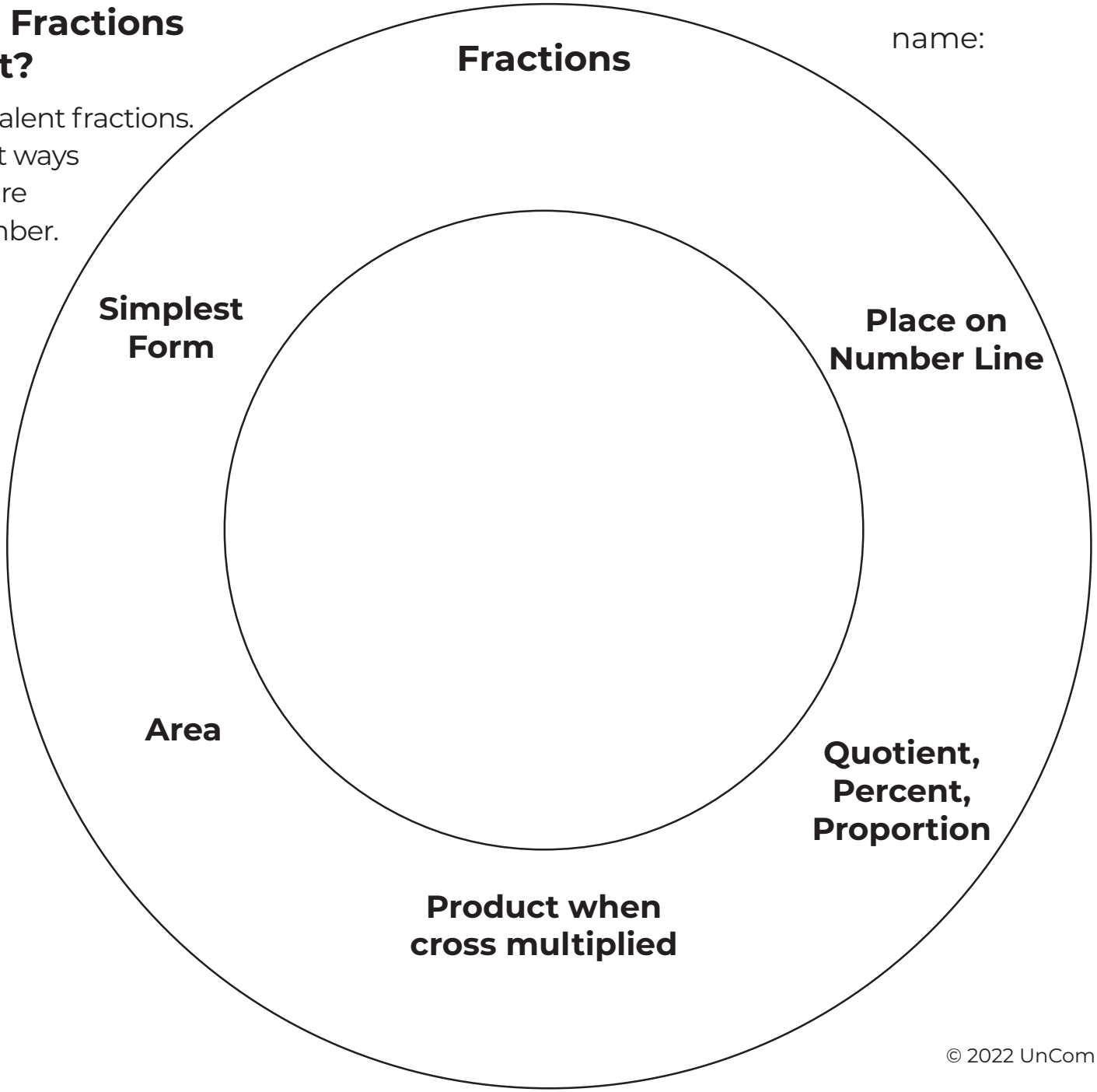
Have your students explain why the fractions are equivalent or why they are not.

	Let your students know what information you want them to include when comparing fractions; simplest form, area, quotient, percent, or proportion. You might start out with only two fractions in order to make the task easier.
	Percent means 'for every hundred.' What number multiplied times the denominator will equal 100? Multiply the numerator by the same amount to find the percent.
	In order to complete this page, students should be familiar with finding the simplest forms of fractions, placing fractions on number lines, drawing pictures to model fractions, thinking of fractions as division problems, cross multiplication and thinking of fractions as numbers that represent proportions.

Are These Fractions Equivalent?

Think of equivalent fractions. Show different ways to see if they are the same number.

name:



Are These Fractions Equivalent?

name

Reflecting on Equivalent Fractions

After students have done a significant amount of work with equivalent fractions, they need a chance to reflect on what they have learned. This allows them to organize the information, make connections, and identify any lingering questions they may still have about the content.

Talking and writing about what they have learned are two effective ways to let students process new concepts.

In order to assess students' progress toward learning goals, teachers are interested in how naturally students use academic vocabulary, how well they understand when the concepts are presented in new contexts, and whether or not the students describe the concepts accurately with lots of detail and examples.

Reflecting on Equivalent Fractions

Discussion questions, writing prompts, and answer key.

How many equivalent fractions are there for one-half?

There are an infinity of fractions equivalent to any number.

How do you know when two fractions are equivalent? List ways to know when fractions are equivalent.

Equivalent fractions share a place on the number line, they have the same decimal, the same simplest form, the same proportion, the same ratio, and the same area. When cross multiplied, the products are the same.

Why do equivalent fractions share the same place on the number line?

Equivalent fractions are the same number. Each number only has one place on the number line.

How can you tell when a fraction has been reduced to its simplest form?

The numerator and denominator have no common factors other than one.

Find five fractions that are equivalent to each other.

Answers will vary.

What happens to a number when it is multiplied or divided by one?

Absolutely nothing. A number does not change when multiplied or divided by one.

What happens when the denominator and the numerator are multiplied by the same number?

The fraction will not change because multiplying the denominator and the numerator by the same number is multiplying the fraction by one, which does not change a number.

Equivalent Fraction Vocabulary Match

name:

Draw a line from the clue to the matching vocabulary word.

1. In simplest form there is a one in the numerator.
2. In simplest form there is a one in the denominator.
3. The numerator and denominator are the same.
4. The numerator is less than the denominator.
5. The numerator is greater than the denominator.
6. The numerator and denominator are the smallest pair of integers with that proportion.
7. This part of the fraction counts how many items are described.
8. This fractional part shows how many equal pieces are needed to make one whole.
9. Fractions that share the same place on a number line.

A. Proper fraction

B. Numerator

C. Whole number in fractional form

D. Simplest form

E. Denominator

F. Equivalent fractions

G. Unit fraction

H. Improper fraction

I. Fraction equivalent to one

Equivalent Fraction Vocabulary Match

Answer Key

1. In simplest form there is a one in the numerator. — G. Unit fraction
2. In simplest form there is a one in the denominator. — C. Whole number in fractional form
3. The numerator and denominator are the same. — I. Fraction equivalent to one
4. The numerator is less than the denominator. — A. Proper fraction
5. The numerator is greater than the denominator. — H. Improper fraction
6. The numerator and denominator are the smallest pair of integers with that proportion. — D. Simplest form
7. This part of the fraction counts how many items are described. — B. Numerator
8. This fractional part shows how many equal pieces are needed to make one whole. — E. Denominator
9. Fractions that share the same place on a number line. — F. Equivalent fractions

Multiply to Find Equivalent Fractions:

name:

X	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$

Multiply to Find Equivalent Fractions:

name:

X										
$\frac{1}{1}$										
$\frac{2}{2}$										
$\frac{3}{3}$										
$\frac{4}{4}$										
$\frac{5}{5}$										

Multiply to Find Equivalent Fractions:

name:

X										

Resources

Teachers

Annenberg Learner Ordering Fractions

Common Core State Standards Initiative Math

NRICH Mathematical Resources

Top Drawer Teachers Modeling Fractions

UnCommon-Core.com

Wild Maths Folding Fractions

Students

Dorling Kindersley DK Find Out!

NCTM Classroom Resources

PBS Learning Media Comparing Fractions

PBS Learning Media Modeling Fractions

UnCommon-Core.com helps teachers explore effective instructional methods and strategies to use in class.

Academic standards organizations define the learning goals that teachers use in class.

UnCommon-Core.com is not affiliated with any academic standard initiative.

A Fresh Look at Equivalent Fractions

Students who can multiply fractions are ready to take
A Fresh Look at Equivalent Fractions.

This series of straightforward activities based on the *multiplicative identity property of one* helps students discover the fundamental structure at the heart of equivalent fractions and grasp why the process of reducing fractions to their simplest form requires the search for common factors.

In this series of activities,
your students will gain experience with:

- unit fractions,
- proper fractions,
- whole numbers written as fractions,
- improper fractions,
- fractions in simplest form,
- fractions equivalent to one,
- equivalent fractions, and
- using academic vocabulary.