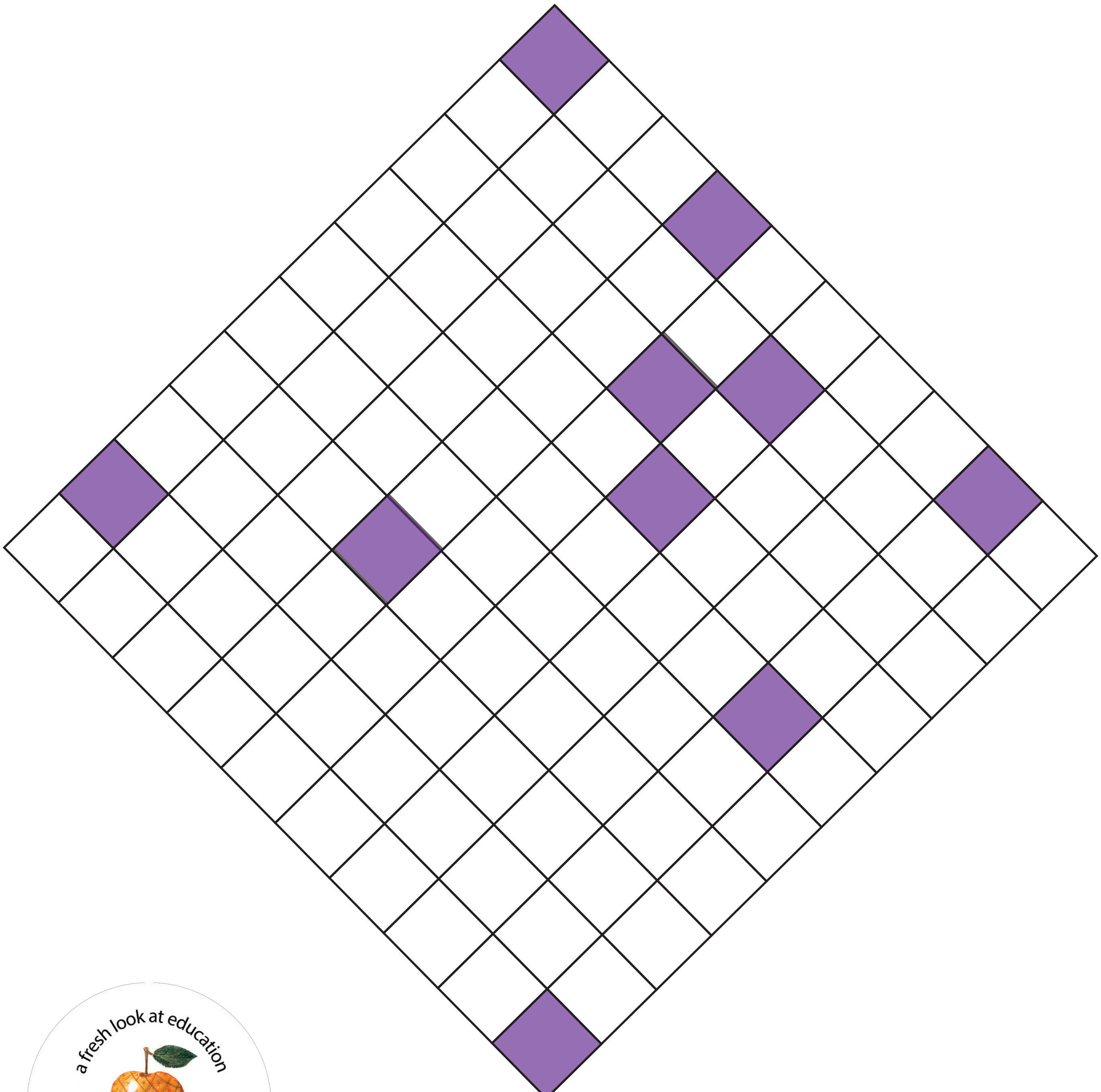
Square Mysteries



a fresh look at education
UnCommon-Core.com

Isabelle Hoag M. Ed.
Director of Education
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Hello Teachers,

Thank you for downloading this handout. After decades of teaching, now I am sharing some of the activities I designed for my students and some new ones as well.

Please, check out the self-paced teacher education courses on UnCommon-Core.com.

While you are there, sign up for your free copy of Colorful Collections: *A Mindful Exploration of Proper Fractions*.

Also, visit my Teachers Pay Teachers store UnCommon-Core dot com.

Thank you again. All the best,

Isabelle

Isabelle Hoag M.Ed.
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Square Numbers

Waves toss seashells onto a beach. Before opening her store, Sally puts them into a three by five array. There are exactly enough places for each of the fifteen shells until:

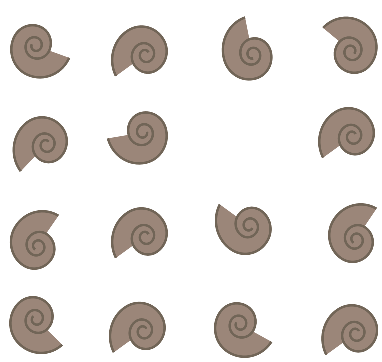
Sally sees another seashell wash ashore.

Sixteen is called a square number due to the shape of the four by four array that can be made from 16 items. Like other figurate numbers, such as triangular and cube numbers, square numbers are named for the shape that they can make.

A square number is the product of a number times itself. They are unique in several ways that are worth investigating. For example, while most natural numbers have an even number of divisors, square numbers have an odd number of divisors because one of them, the square root, is used twice.

Running diagonally through any multiplication table, the series of square numbers provides fun geometric patterns and interesting numerical relationships to study.

Enjoy.



Dear Teachers,

Square Mysteries gives students who can multiply and find square numbers lots of math practice. This booklet includes five investigations, one of which is still an open question in math. While unraveling these mysteries, your students will add, subtract, multiply, and find square numbers. They will consider patterns found in series of numbers such as: consecutive integers, odd numbers, and square numbers. They will also review mathematical vocabulary and use tables to clarify the underlying mathematical patterns.

Hopefully, **Square Mysteries** will grab children by their imaginations and never let go. Your students will become life long math lovers; always searching for interesting numerical relationships. Along with their interest, their number sense and mathematical proficiency will soar to unexplored heights. A teacher can dream.

Lastly, I hope these mysteries capture your own intellectual curiousity. In which case, here's a chance to share your enthusiasm for mathematical exploration with your students.

Enjoy,

Isabelle

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Number Sense

Messing about with the awesome number patterns found in and around square numbers provides students with opportunities to relax and enjoy some wild numerical relationships. These are the perfect conditions for growing their instinctive understanding of numbers and giving them a chance to experience doing some real math. As they uncover the number patterns at the heart of these **Square Mysteries** students' excitement will grow; as will their number sense. Invite your students to add more ideas and information to the Square Number Mind Map after each set of activities. Reflection will help strengthen their number sense.

Number sense is that elusive body of knowledge that accumulates as we discover more about the underlying properties of and relationships between numbers. It tells us when a number is even or odd or when a larger number might be a multiple of a smaller number. The wonderful story about how Taxi cab numbers got their name is an example of Srinivasa Ramanujan's amazing number sense. It was said that every positive integer was one of his personal friends.

Understanding the numeric patterns hidden in the mysteries is not directly included in math standards and should not be used as a learning goal. The skills and concepts your students develop while investigating **Square Mysteries** are included in academic standards.

Learning goals addressed in **Square Mysteries** may include ideas such as: using academic vocabulary correctly, calculating accurately, or studying square numbers. In order to test the validity of the **Square Mysteries** students have to perform several calculations, track and evaluate the results to ensure all the answers are correct, and then make conclusions based on their findings. Any miscalculation will obscure the pattern. Your students will be very interested in solving the problems accurately. Successful completion of these activities will also strengthen your students' academic vocabularies, logical thinking, and confidence.

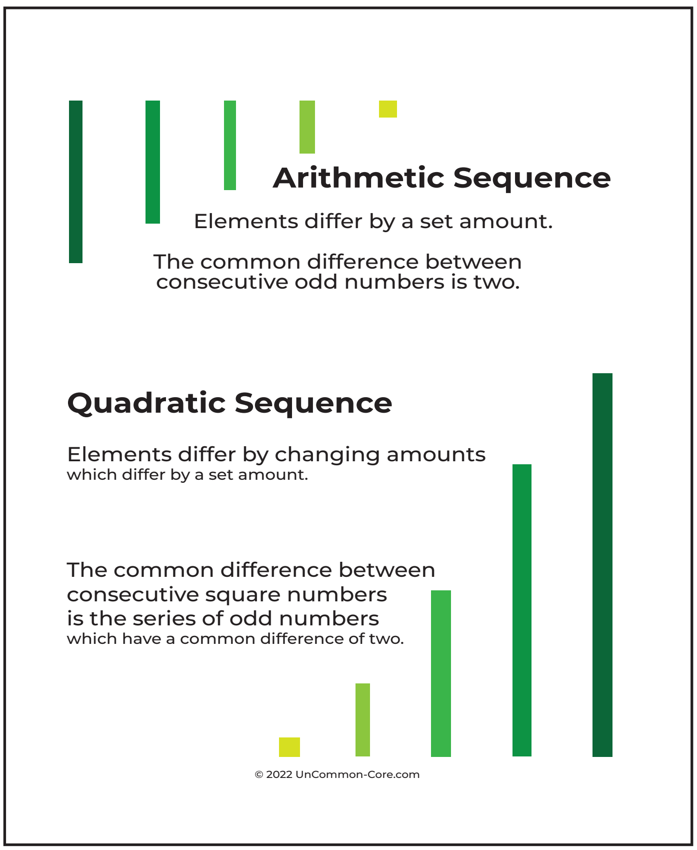
Real mathematicians often collaborate with their peers. These activities are tailor-made for small group or whole class teamwork.

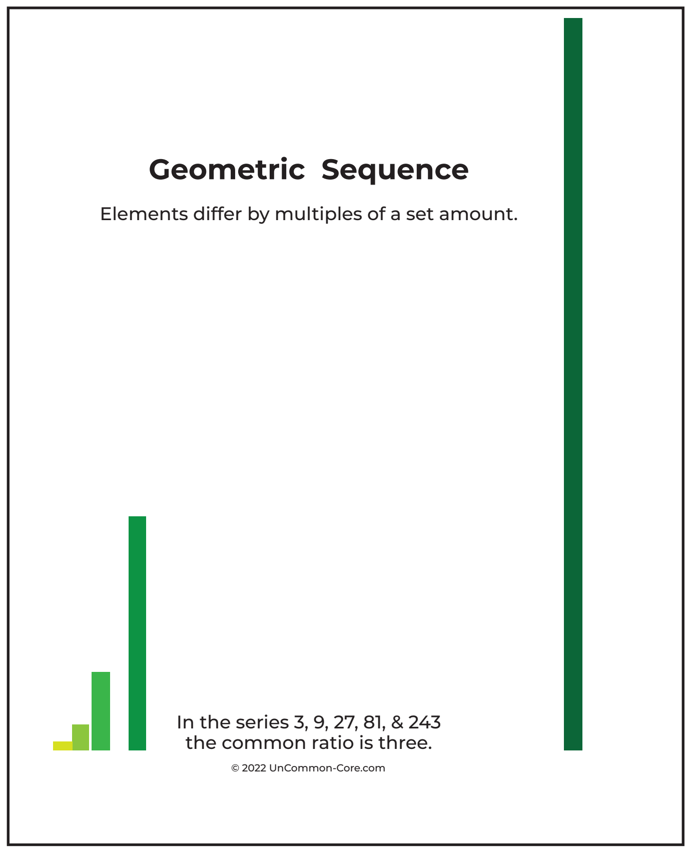
Number Patterns

It is human nature to collect patterns. Our brains love hidden relationships and repeated motifs. Learning how to sort patterns by their defining characteristics is an essential academic skill in many subjects.

Having class discussions about types of number patterns before diving into the activities in **Square Mysteries** provides academic context for your students. They will know what to look for when working with patterns. Talking about types of patterns will also support your students' abilities to find connections between subjects such as art, music, science, and math.

Identifying and naming various patterns will also support your students' academic vocabulary. For example, the word quadratic comes from quadratum, the Latin word for square. Similarly, quartus, Latin for fourth, is an ancient ancestor of words such as quarter, quart, and quartet. Teach your students the vocabulary they need in order to be able to compare and contrast different mathematical relationships.





Arithmetic Sequence

Elements differ by a set amount.

The common difference between consecutive odd numbers is two.

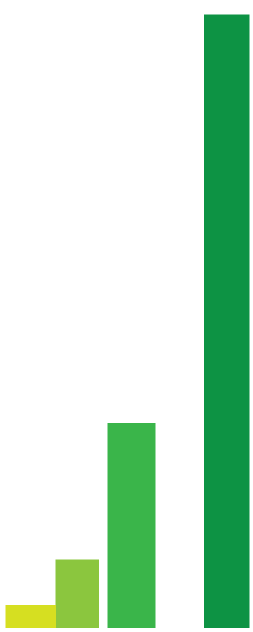
Quadratic Sequence

Elements differ by changing amounts - which differ by a set amount.

The common difference between consecutive square numbers is the series of odd numbers - which have a common difference of two.

Geometric Sequence

Elements differ by multiples of a set amount.



In the series 3, 9, 27, 81, & 243
the common ratio is three.

Teacher Tips

Gather plenty of graph paper, crayons, markers, scissors & paste.
Prepare to have serious fun doing serious math with your students.

Before you start, decide if you would like to make packets of stapled pages or if you will present each mystery independently. In either case make sure to browse through the options carefully. Some of the mysteries have alternative handouts offering differing amounts of support to students.

Review academic vocabulary such as 'consecutive', 'series', or 'number pattern' before starting the activity. Students might need to revisit more familiar vocabulary such as 'sum', 'difference', or 'square number' in order to make sure they understand the exact mathematical meaning of these terms. Until a student uses a vocabulary word naturally in speech and writing, they do not 'own' that word.

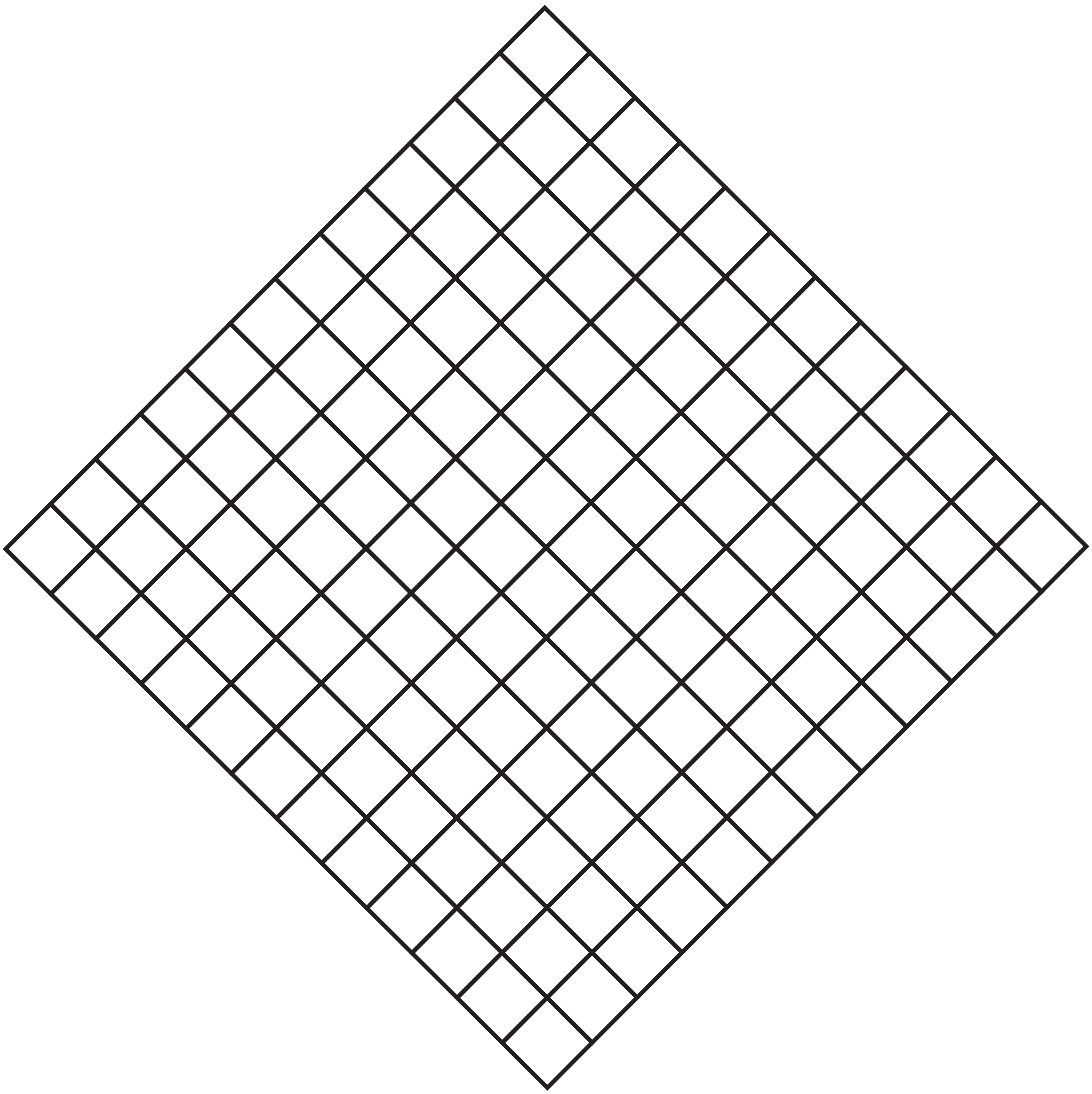
Be on the lookout for junior mathematicians. These activities are not strict mathematical proofs. They merely wave in the direction of some numerical properties. Some students may be unsure whether or not the number patterns revealed in **Square Mysteries** hold true for numbers of larger magnitudes.
This hesitancy signals that your student is thinking like a real mathematician.
You might want to let them know that if they still have these questions by the time they get to college, they should consider taking some math classes to help them prove whether or not the patterns continue as the numbers grow.

If you want to capture samples of your students' thinking as they work, you could have some sticky notes available for them to jot down questions and observations. Collect these gems on the whiteboard or a sheet of paper with 'Questions and Comments' as a title. Then, once the activities are completed, revisit the messages on the sticky notes during class discussion.

Delving into the **Square Mysteries** is your chance to model academic language, logical thinking, and problem solving strategies in a less formal setting than a typical math class. Let your enthusiasm shine as you and your students uncover patterns hidden in and between square numbers.

UnCommon-Core.com helps teachers explore instructional methods and best practices to use in class. Common Core State Standards define the content teachers use in class. The two organizations are independent of each other.

Square Mysteries



The First Mystery

Yes, the series of odd numbers is hidden in between consecutive square numbers! In this activity students subtract to find the difference between consecutive square numbers revealing the series of odd numbers in between each square number. *Seriously, how cool is that?*

The numbers running along the top of the grid on the handout are regular counting numbers. The rows are defined by math problems in which each square number is subtracted from the next larger square. Once completed, the grid reveals the series of odd numbers, an arithmetic sequence with a common difference of two.

When introducing this to your class, make sure to discuss how miscalculations might affect the overall pattern. For instance, if they goof up and record 64-49 as 14 there will be a break in the stair step appearance of the final design.

Eagle-eyed students will point out that an even number minus an odd number must be another odd number. As students finish the activity you may want to invite them to write about their experiences. Encourage them to use what they know about combining even and odd numbers in order to understand why the difference between squares must always be an odd number.

Mathematicians think of the relationship like this:

$$n^2 - (n-1)^2 = 2n-1$$

$$8^2 - (8-1)^2 = 2 \times 8 - 1$$
$$64 - 7^2 = 16 - 1$$
$$64 - 49 = 15$$

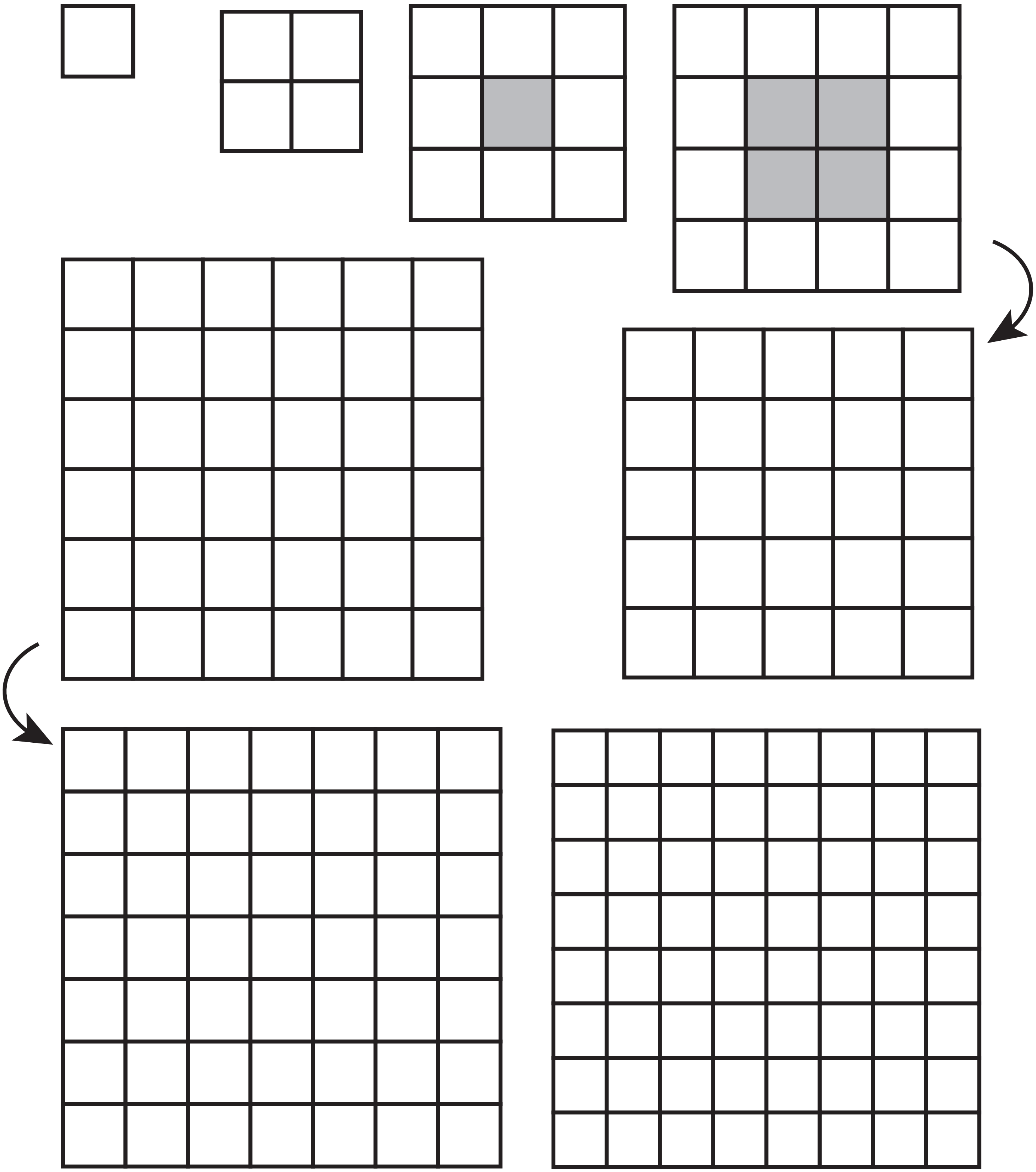
The first part of that equation tells us we are talking about the difference between squares of consecutive whole numbers. The second part defines the amount of the difference. The right side of the equation represents an odd number; any even number 2n minus 1 will be odd.

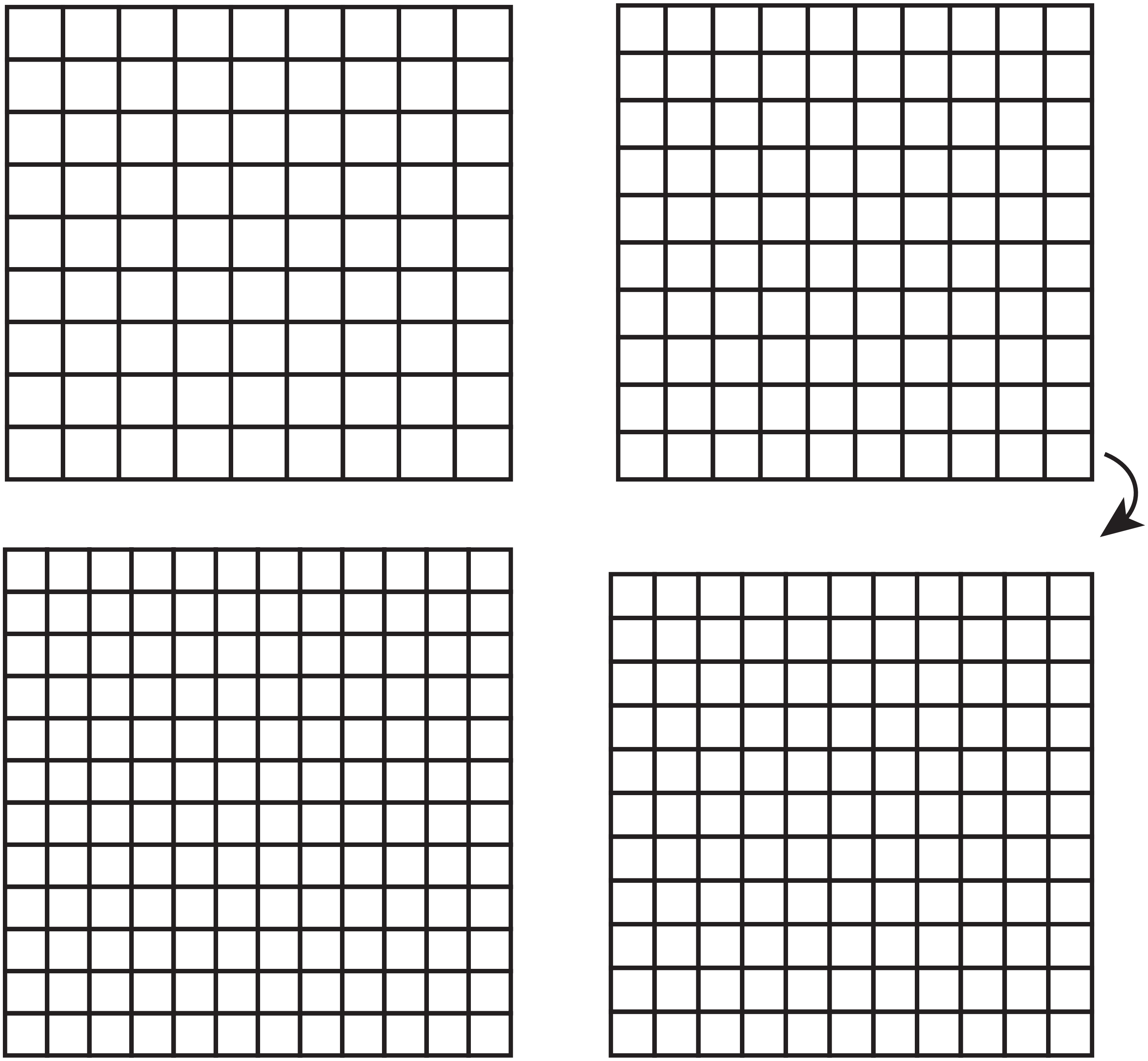
When *n* is 8 we see that 64-49 is 15. Since 8+7 is also 15, the sum of the consecutive numbers equals the difference of their squares.

There are three versions of this handout, offering three levels of support. If you can't choose which one best fits your class, why not make copies of all three and let each student decide which they want to use. Like all **Square Mysteries**, this activity is accessible to beginners and yet offers enough complexity to interest more advanced students.

Squares Inside Squares Inside Squares

Color the squares centered inside other squares.





What did you notice about squares inside squares?

The First Mystery:
A Pattern Between the Square Numbers

Fill in the chart below. Find the difference between each square number and the one after it. What pattern do you see?

ODD NUMBERS BETWEEN SQUARES

THE SEQUENCE OF ODD NUMBERS

SUBTRACTING EACH SQUARE FROM THE NEXT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1-0																							
4-1																							
9-4																							
16-9																							
25-16																							
36-25																							
49-36																							
64-49																							
81-64																							
100-81																							
121-100																							
144-121																							

Describe the pattern found in between consecutive square numbers.

Explain what you were thinking as the pattern came into focus.

When do you think this pattern will end? How so?

The First Mystery:
A Pattern Between the Square Numbers

Use the chart below to show the differences between consecutive square numbers.
Then write a descriptive title and labels. The first two rows have been done for you.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1-0				(1x1)-(0x0)																			
4-1				(2x2)-(1x1)																			
9-4																							
16-9																							
25-16																							
36-25																							
49-36																							
64-49																							
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1-0				(1x1)-(0x0)																			
4-1				(2x2)-(1x1)																			
9-4																							
16-9																							
25-16																							
36-25																							
49-36																							
64-49																							
81-64																							
100-81																							
121-100																							
144-121																							

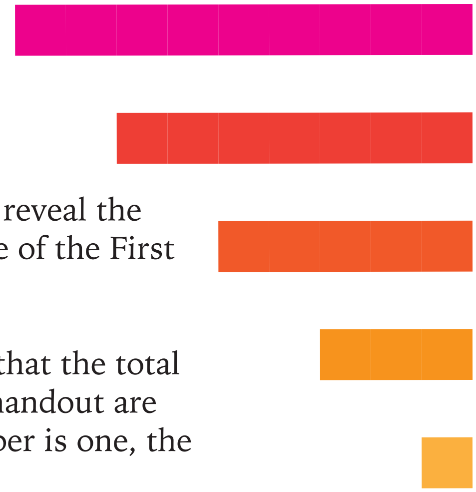
Describe the pattern found in between consecutive square numbers.

Using what you know about even and odd numbers, explain why the difference between consecutive square numbers is always an odd number.

Square Number Mind Map

Show what you know
about square numbers

The Second Mystery



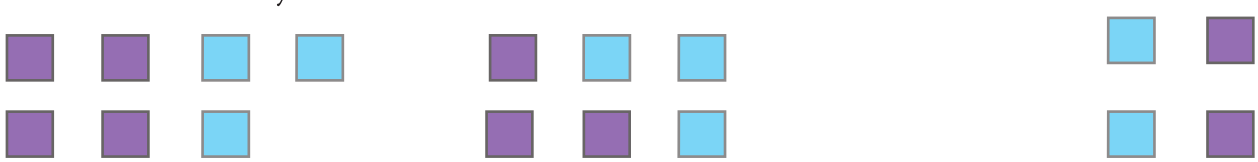
Yes, adding consecutive odd numbers, starting with one, will reveal the series of square numbers. The Second Mystery is the flip side of the First Mystery. *Shhhhh!*

In this activity, students add successive odd numbers to find that the total is always a square. Running along the top of the grid on the handout are the ordinal positions of each odd number. The first odd number is one, the second odd number is three, the third is five and so on.

Along the side is the series of square numbers. Each square number is also the total of the odd numbers in that row. In the second row, the sum of the first two odd numbers is four which is two times two. In the third row, the sum of the first three odd numbers is nine which is three times three. Once completed the handout reveals the quadratic sequence of square numbers which has a common difference equal to the series of odd numbers.

An interesting twist in this mystery is that the ordinal position of the final odd number added is also the root of the resulting square. The ordinal position of the final addend is the root of the corresponding square number.

As students finish the activity you may want to invite them to write about their experiences. Encourage them to use what they know about combining even and odd numbers in order to figure out why the square numbers follow an even/odd pattern. You may have to point out that the sum of an odd number of odd numbers will be odd. The sum of an even number of odd numbers will always be even.



When introducing this activity to your class, make sure to discuss how miscalculations might impact the overall pattern. For instance, if they goof up and get 17 when they add 1+3+5+7 what will happen? Encourage them to see how they can use the bigger pattern to check their work. In this case, since the previous square number is nine, which is an odd number, they should expect the next square number will be an even number, so a total of 17 must be wrong.

Students might want to make a larger version of the chart in order to see the pattern better.

There are three versions of this handout, offering three levels of support. There are also coloring pages that illustrate the same ideas which might be a good source of support.

Square Numbers and Multiplication

Multiply to find the products.
Decorate the grid
to show the square numbers.

times	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Square Numbers and Prime Numbers 10 x 10

Write in the numbers.
Decorate the grid to show square and prime numbers. Give it a title.

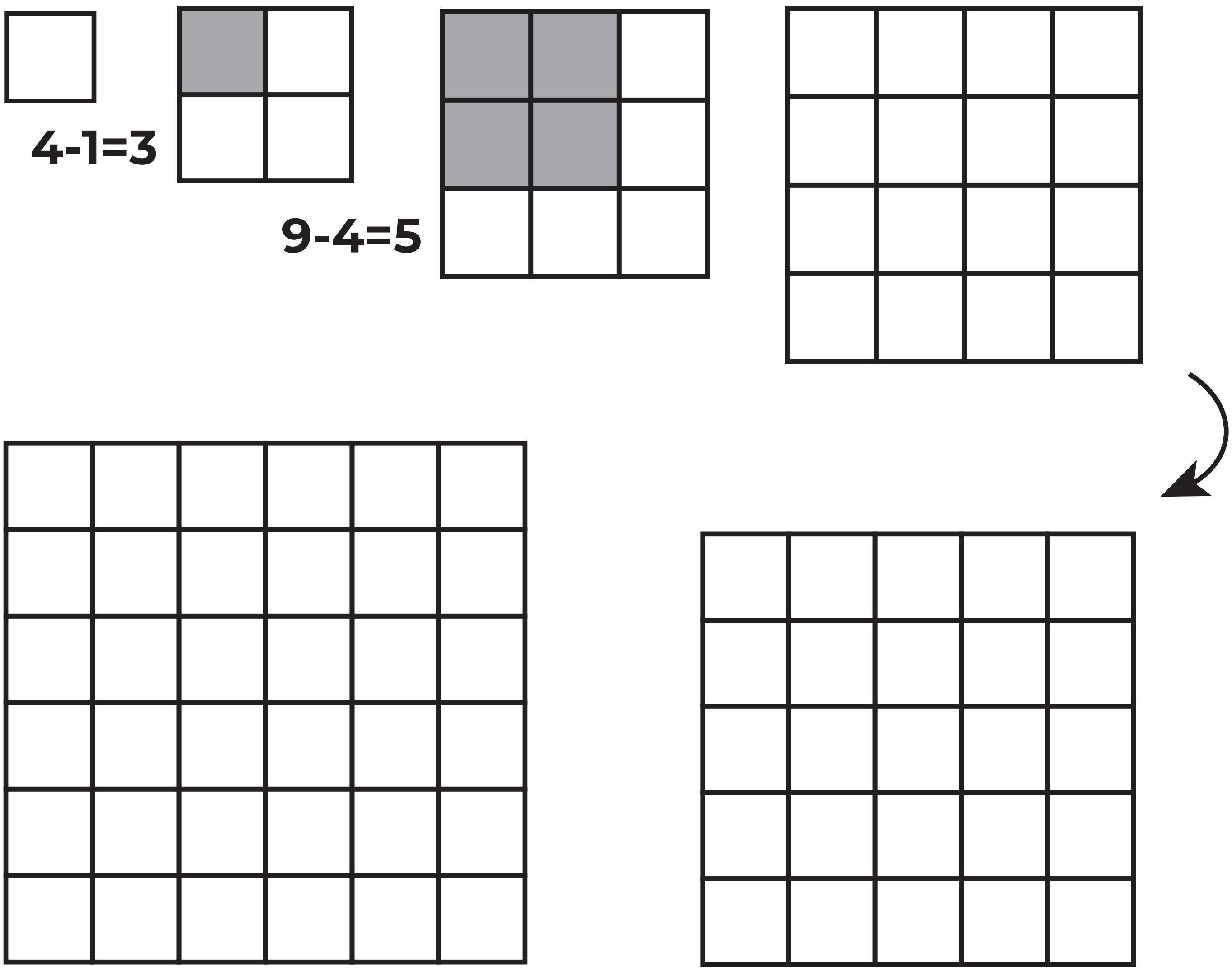
Square Numbers and Prime Numbers 12 x 12

Write the series of whole numbers in the grid. Color in specific cells to show square and prime numbers. Give it a title.

A Fresh Look at The Square Mysteries

Color the same number of cells found in the small square in the next larger square.

Write a number sentence to show how many more cells are in the larger square.



How does this illustration help explain the Square Mysteries?

The Second Mystery:
What Happens when We Add Consecutive Odd Numbers?

Complete the chart below. What patterns do you see?

Add Odd Numbers in order: Find Squares!

How many places are in this line?

The series of square numbers follows an even/odd pattern.

Seven is the 4th odd number in line.

The square numbers are also the sums of the numbers in that row.

The Odd Numbers' place in line

The Series of Square Numbers	1st	2nd	3rd	4th	5th	6th
1	1					
4	1	3				
9	1	3	5			

Use another page to make a larger chart or
an illustration like the one on the back cover to answer the question.

What happens when you add consecutive odd numbers?

Please describe the pattern that you see:

The Second Mystery:
Can Adding Consecutive Odd Numbers Reveal the Square Numbers?

Complete the chart below. Include a descriptive title.

Will nine plus seven equal 4 x 4?

Hey, these sums look familiar. Have I seen this series before?

Does this mean that the sum of the first **ten** odd numbers equals **10 x 10**?

Is **twelve squared** equal to the sum of the first **twelve** odd numbers?

The first *two* odd numbers are 1 & 3. The sum of one and three is four. Four is two times two!

CONSECUTIVE ODD NUMBERS

SUMS OF CONSECUTIVE ODD NUMBERS	1ST	2ND	3RD	4TH	5TH	6TH
1	1					
4	1	3				
9	1	3	5			

Use another page to make a larger chart to answer the question.

How are the sums of consecutive odd numbers related to the series of square numbers?

Please describe the pattern you see:

The Second Mystery:
Can Adding Consecutive Odd Numbers Reveal the Square Numbers?

Complete the chart below. Include a descriptive title.

Hey, these sums look familiar. Have I seen this series before?

Will nine plus seven equal 4 x 4?

Does this mean that the sum of the first **ten** odd numbers equals **10 x 10**?

Is **twelve squared** equal to the sum of the first **twelve** odd numbers?

	1st	2nd	3rd	4th	5th	6th
1	1					
4	1	3				
9	1	3	5			

The first *two* odd numbers are 1 & 3. The sum of one and three is four. Four is two times two!

Choose from the writing prompts below. Share your response on another paper.

Is the sum of consecutive odd numbers always equal to a square number? Why do you think so?

Write some equations to see if adding sequential odd numbers (starting from one) will always result in a square number. What did you discover?

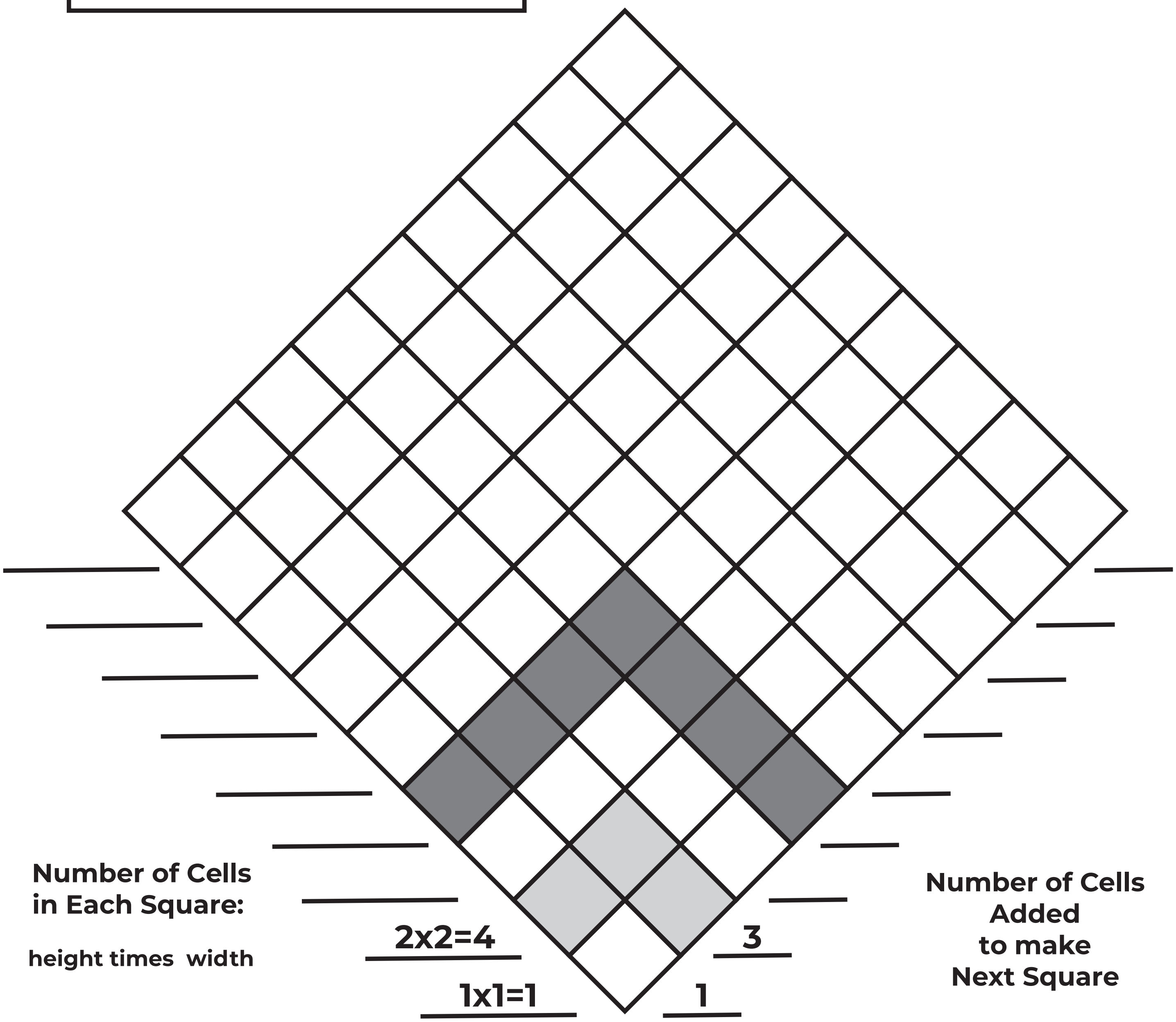
How is the ordinal position of the last odd number in the addition sentence related to the square number created?

Copy then add on to the illustration on the back cover. Describe how that illustration is related to The Second Mystery.

Compare and contrast the first and second square mysteries.

Another Look at The Second Square Mystery

Color the cells needed to make the next larger square. Write the number of cells colored on the right and the height by width on the left.



How does this illustration help explain the Square Mysteries?

The Third Mystery

Yes, the difference between consecutive square numbers is equal to the sum of their roots. From another point of view: the sum of a pair of consecutive whole numbers is the same number as the difference of their squares.

From the first two mysteries, your students are now pros at finding the difference between pairs of consecutive square numbers. You may want to have them practice adding pairs of consecutive whole numbers in advance of starting on this activity. The handout Adding Pairs of Consecutive Whole Numbers will provide lots of mindful math practice for your students. It shows another way to visualize the sequence of odd numbers.

Yes, both adding consecutive numbers and subtracting consecutive squares will result in the series of odd numbers. *Hmmmm.*

In order to make sense of this activity, students must understand the vocabulary involved. The sentences in math texts are dense. They are packed with exact terms. The definition of each mathematical term informs and contributes to the overall meaning of the text.

To complete this activity, students must know exactly how ‘sum’, ‘difference’, ‘consecutive’, and ‘square number’ are used in math. They must know what whole numbers are. They have to understand that ‘square’ can be used as a noun and a verb. They have to be able to translate ‘related to’ as a choice among ‘equal to’, ‘greater than’, or ‘less than’. Other vocabulary words that students need to know include: even, odd, pattern, product, multiplied, grid, cell, row, column, ‘how many more’, subtract, smaller, larger, substitute, pair, and ‘the same’.

Once they get the big picture, your students might want to sort out why this relationship works. From the first two mysteries, they know that the difference between any pair of sequential square numbers must be an odd number. Also, they can figure out that a pair of consecutive whole numbers will contain an odd and an even number. Combining an odd number with an even number yields an odd number for an answer.

So the remaining mystery is why the magnitude of the sum and the difference between the squares of consecutive whole numbers is the same. Think of three sets of numbers: the set of consecutive pairs of whole numbers, the set of sequential odd numbers and the set of consecutive pairs of square numbers. Line them up side by side and you will see that the first elements of each set naturally correspond to each other.

If you invite your class to create some sort of representation to describe the relationship in this mystery, please decide in advance what sort of display they could make. You could share the mini posters to give your students some ideas for their work. You could also share a copy of the Sample Square Number Mind Map shown on the resources page. Working the ideas they have just discovered into a graphic display or work of art provides students with an important chance to reflect on what they have learned.

Adding Pairs of Consecutive Whole Numbers

Use mental math
to find the sums below.

1+2=	13+14=	25+26=
2+3=	14+15=	26+27=
3+4=	15+16=	27+28=
4+5=	16+17=	28+29=
5+6=	17+18=	29+30=
6+7=	18+19=	30+31=
7+8=	19+20=	31+32=
8+9=	20+21=	32+33=
9+10=	21+22=	33+34=
10+11=	22+23=	34+35=
11+12=	23+24=	35+36=
12+13=	24+25=	36+37=

Questions to think about:

What patterns did you find while adding pairs of consecutive whole numbers?

What pattern would you find if you subtracted pairs of consecutive whole numbers?

Where else have you seen this pattern?

How do patterns make math easier?

The Third Mystery:

The actual statement of this mystery is too exciting to share with you at the present time. We apologise for any inconvenience this may cause. The question will be revealed later. Meanwhile, the first one has been done for you.

Choose two consecutive whole numbers.

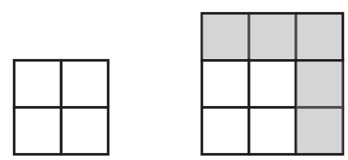
2, 3

Two and three are consecutive numbers: They are next to each other on the number line; one right after the other. There aren't any other whole numbers between them. They follow the even/odd pattern expected of consecutive whole numbers.

Next, square the consecutive numbers.

Four and nine are the squares of two and three. Perfect squares are the products of an integer multiplied by itself. Two times itself is four. Three times itself is nine.

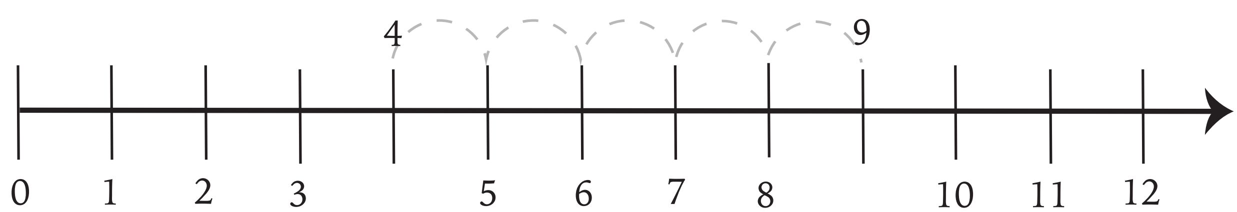
Square numbers can be arranged in grids that have the same number of rows as columns. A grid with four cells can have two rows and two columns. A grid with nine cells can have three rows and three columns. A four by four grid will have 16 cells.



Now, find the difference between the squares of the consecutive numbers.

One way to find the difference between four and nine is to count how many more cells the 2 x 2 grid would need in order to have the same amount of cells as the 3 x 3 grid. Otherwise, subtract the smaller number from the larger one, or count the jumps between four and nine on the number line.

The difference between two times two and three times three is five.



Restate the question. Substitute numbers for phrases in the question.

How is the sum of two consecutive whole numbers related to the difference between the squares of those two numbers?

Two and three are a pair of consecutive numbers:
How is the sum of **two plus three** related to the difference between their squares?

Four and nine are the squares of two and three:
How is the sum of 2+3 related to the difference between **four and nine**?

The difference between four and nine is **five**:
How is **the sum of 2+3** related to the difference between their squares which **is 5**?

You have got to be kidding me.
How is 2+3 related to 5 **which is also the difference between their squares?**

Related? It is the SAME NUMBER! It's equal! Is this for real? Who else knows about this?

When the consecutive numbers are two and three:

Their sum
IS EQUAL TO
the difference between their squares.

Cosmic Coincidence?

Is this some strange property of two and three, or do other consecutive numbers share this mind-bending relationship? How will you know for sure (without asking Siri or Alexa)? What kind of evidence will it take to make up your mind? Would it be easiest to test one pair of numbers at a time (until you get the hang of it) and then make some sort of chart? Will there be enough space for more leading quest -

The Third Mystery: Your Turn
Is the Sum of Two Consecutive Whole Numbers Equal to the Difference Between their Squares?

First, choose a pair of consecutive whole numbers.

Next, square the consecutive numbers.

Now, find the difference between the square numbers.

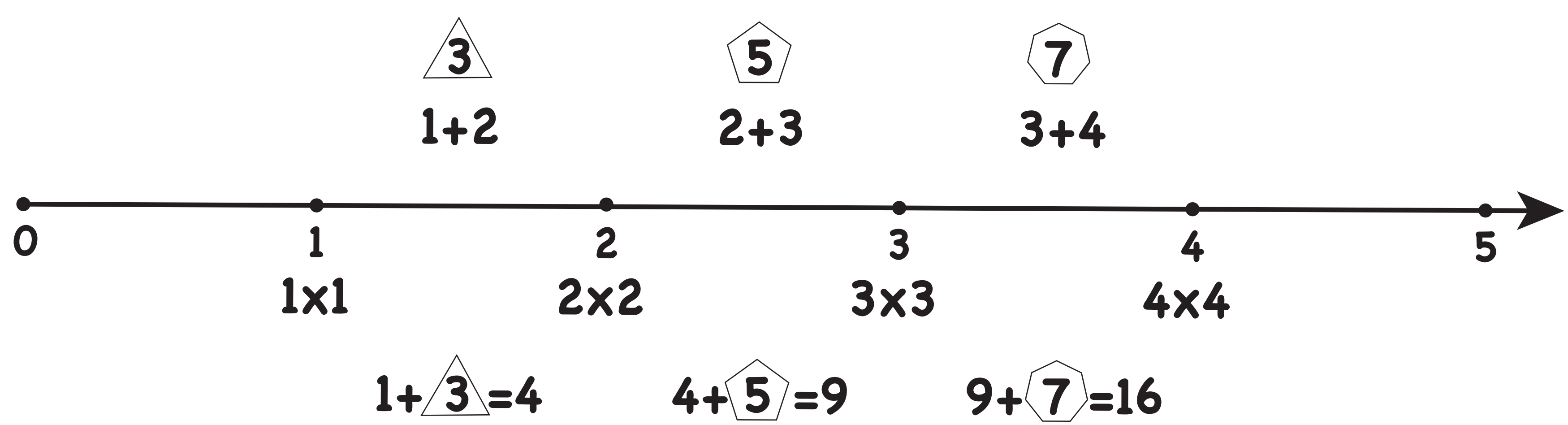
Restate the question.
Substitute actual numbers for phrases in the question.

How does the difference between their squares relate to the sum of two consecutive whole numbers? = <>

Is this a property of all whole numbers, serendipity, or just a few strange coincidences? Test more pairs of consecutive numbers to help you decide. Share your thoughts here:

You might want to create a chart that shows what is going on.

Sums of Consecutive Numbers
Equal
Differences Between their Squares



The sum of consecutive whole numbers

One times One is One

Two times Two is Four

$$1+2=3=4-1$$

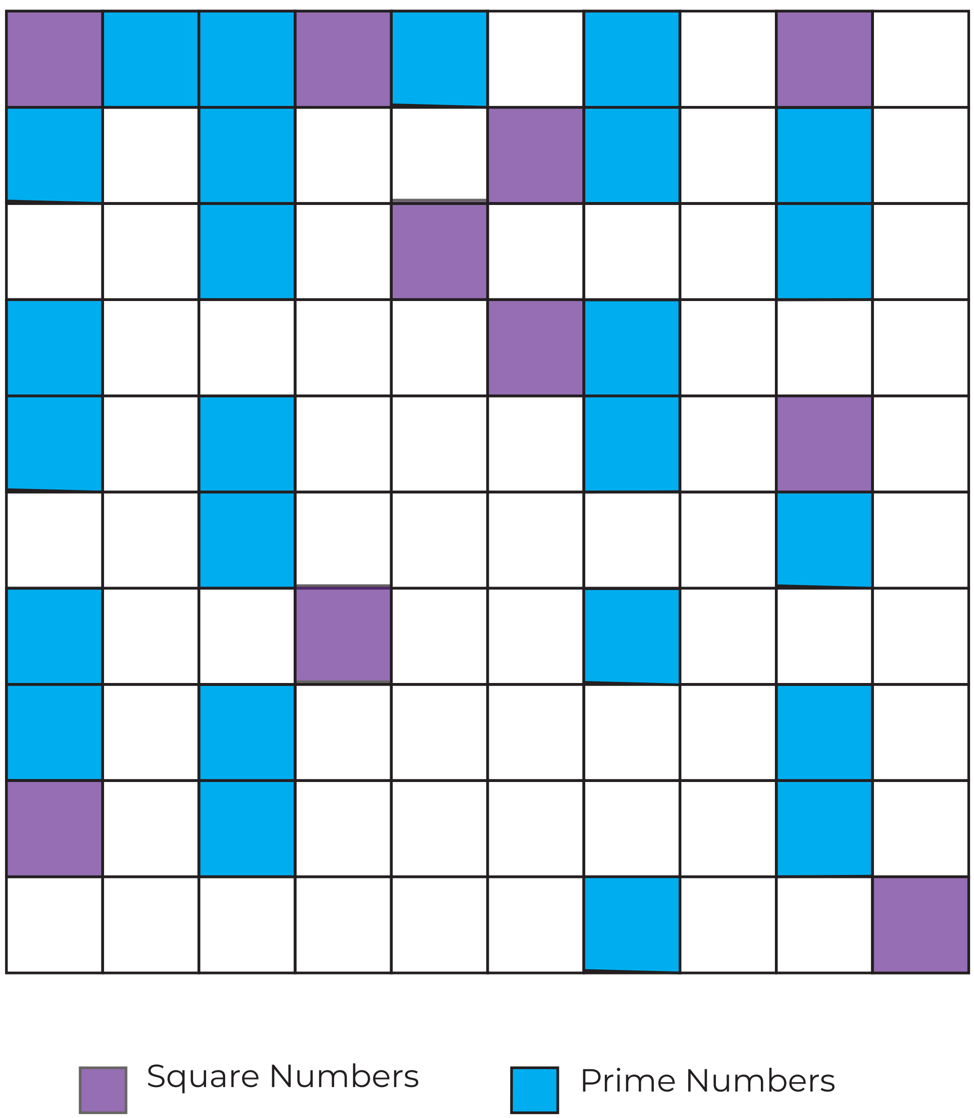
is the difference between their squares.

Another Mystery
Legendre’s Conjecture

Is there a prime number between each pair of consecutive squares?

This problem was first posed by Adrien-Marie Legendre in 1798. He thought the answer was ‘yes’, however he couldn’t prove it for certain. As it turns out, neither could anyone else. Thus, **Legendre’s Conjecture** remains an open question in mathematics.

While elementary students do not have the skills to prove if it is true or not, they can appreciate the question and think of various ways to approach it. The process of investigating Legendre’s Conjecture requires logical thinking, multi-step problem solving, and lots of math practice.



Another Mystery:

Is there a Prime Number Between Each Pair of Consecutive Squares?

In 1798 Adrien-Marie Legendre asked:

Does each pair of consecutive square numbers have at least one prime number between them?

No one has figured it out yet! Just find one pair of consecutive square numbers that does NOT have a prime number between them.

Pairs of Consecutive square numbers	Prime numbers between the pair of square numbers
1 and 4	3
4 and 9	5 and 7

The ‘Is It a Square Number’ Mystery

When it comes to sifting through large numbers in search of perfect squares there are a few tests to identify possible candidates. This activity introduces two of these tests which are comprehensible to young students and offer plenty of math practice.

The first test is simply to look at the number in the unit’s place. Square numbers will never have a 2, 3, 7, or 8 in the unit’s place. Remind your students that even if a large number passes this test, that doesn’t mean it IS a square, it means, the number COULD be a square.

The second test involves a lot of mental math. The sum of the digits of any square number will always add up to one, four, seven, or nine. Again, make sure your students understand what it means when a number passes this test. There are many numbers with digit sums of 1, 4, 7, or 9 that are not perfect squares. This is a great way to teach formal logic skills.

Finding digit sums is a great way for students to practice their mental math strategies, practice number facts, and discover some cool patterns. Encourage your students to make lists of various series of numbers and then find the digit sums of those numbers in their spare time, yes, for fun.

Students can test their sleuthing skills in the Could this Number be Square activity. From a list of large numbers, they have to identify potential square numbers and then determine which of the candidates match the roots given at the top of the page.

Fun fact: if the digit sum of a number is divisible by three then that number is also divisible by three. This must be why there is only one prime number with a digit sum of three.

Times 3	Digit Sum
3	3
6	6
9	9
12	3
15	6
18	9
21	3

Times 6	Digit Sum
6	6
12	3
18	9
24	6
30	3
36	9
42	6

Times 9	Digit Sum
9	9
18	9
27	9
36	9
45	9
54	9
63	9

Digital Sums

Add the individual digits together to find the digit sum of each integer.

169: 1+6+9=16=1+6 =**7**

Square Numbers	Odd Numbers	Even Numbers
1	1	2
4	3	4
9	5	6
16	7	8
25	9	10
36	11	12
49	13	14
64	15	16
81	17	18
100	19	20
121	21	22
144	23	24
169	25	26
196	27	28
225	29	30
256	31	32
289	33	34
324	35	36
361	37	38
400	39	40

Could this Number be Square?

How can you tell if a really big number is also a square number? Before you reach for a calculator, examine the number closely. There are clues that will let you know which numbers could be squares and which could not. Let’s look at two of them.

First, identify the digit in the unit’s place. The unit’s place, sometimes called the one’s place, is farthest to the right. The digit in this position has the smallest value in the number. If the number in the unit’s place is even, the entire number is even. If this number is odd, then the entire number is odd.

Square numbers might have a zero, one, four, five, six, or nine in the unit’s place. Square numbers will never end with a two, three, seven, or eight. So, looking at the digit in the unit’s place will give you the first clue as to whether or not the large number might be square.

Square Root	**Square Number**	**Unit’s Place** not 2, 3, 7, 8	**Digit Sum** 1, 4, 7, 9	**Unit’s Digit Connection**
8	**64**			
9	**81**			
10	**100**			
11	**121**			
12	**144**			

Next, add the individual digits in the number. Keep adding until you have a single digit number. This single digit number is called the digit sum. Digit sums can tell you a lot about a number.

The digit sum of a square number will always be one, four, seven, or nine. Of course, there are many other numbers that are not square that will have digit sums of one, four, seven, or nine. However, if a number has a digit sum of two, three, five, six, or eight then it cannot be a square number.

Once you find a number that does not have a two, three, seven, or eight in the unit’s place and has a digit sum of one, four, seven, or nine, then you will have to continue investigating to decide if it is a square number or not.

Unit’s Place Connections between Squares and their Roots	
If the unit’s digit of the root is:	**The square will end with:**
0	00
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	25

Could this number be square?

50,320,014

The digit in the unit’s place is four, like some square numbers.

5+3+2+1+4=15=1+5=**6**

The number’s digit sum is six. After checking a list of eight digit square numbers, it is clear that fifty million, three hundred twenty thousand, and fourteen is in between consecutive square numbers 50,296,464 and 50,310,640. Since there cannot be another square number in the middle of two consecutive squares, 50,320,014 is not a square number.

Only a few of the millions and billions of whole numbers can be square.

Could These Numbers be Square?

Three of the large numbers in the box below are perfect squares. The numbers in this box are their roots. See if you can match the square with its root without using division or multiplication.

140 **1729** **3163**

When you find a match, highlight the square number and its root in the same color.

Large Numbers	Unit's Place not 2, 3, 7, 8	Digit Sum 1, 4, 7, 9	Possible Square?
10,004,569			
85,440			
2,989,441			
500,070,590			
45,807			
75,541			
19,600			
2,310,664			
2,000,000,005			
five million			
798,222			
410983			
152892			
3162946			
100297215			

Explain how you matched the square numbers with their roots.

The Glossary

Cardinal Numbers: numerals that indicate an amount or value

Consecutive: sequential, following each other without a break

Difference: the result of subtraction, the answer to a subtraction problem

Endless Loop: see infinite loop

Even Number: divisible by two with no remainder, unit's place has 0, 2, 4, 6, or 8

Factor: a number that divides another number evenly

Infinite Loop: see endless loop

Integers: whole numbers and negative numbers but not fractions or decimals

Odd Number: has remainder when divided by two, unit's place has 1, 3, 5, 7, or 9

Ordinal Numbers: numerals that indicate order; first, second, third . . .

Parity: a property of integers indicating their even or odd status

Perfect Square: the product of an integer times itself

Prime Number: an integer with no factors other than itself and one

Product: the result of multiplying two factors together

Series: a group of similar items arranged one after the other

Square Number: the product of a number times itself

Sum: the result of addition, the answer to an addition problem, the total

Perfect Squares
1
4
9
16
25
36
49
64
81
100
121
144

Resources

[Common Core State Standards Initiative]

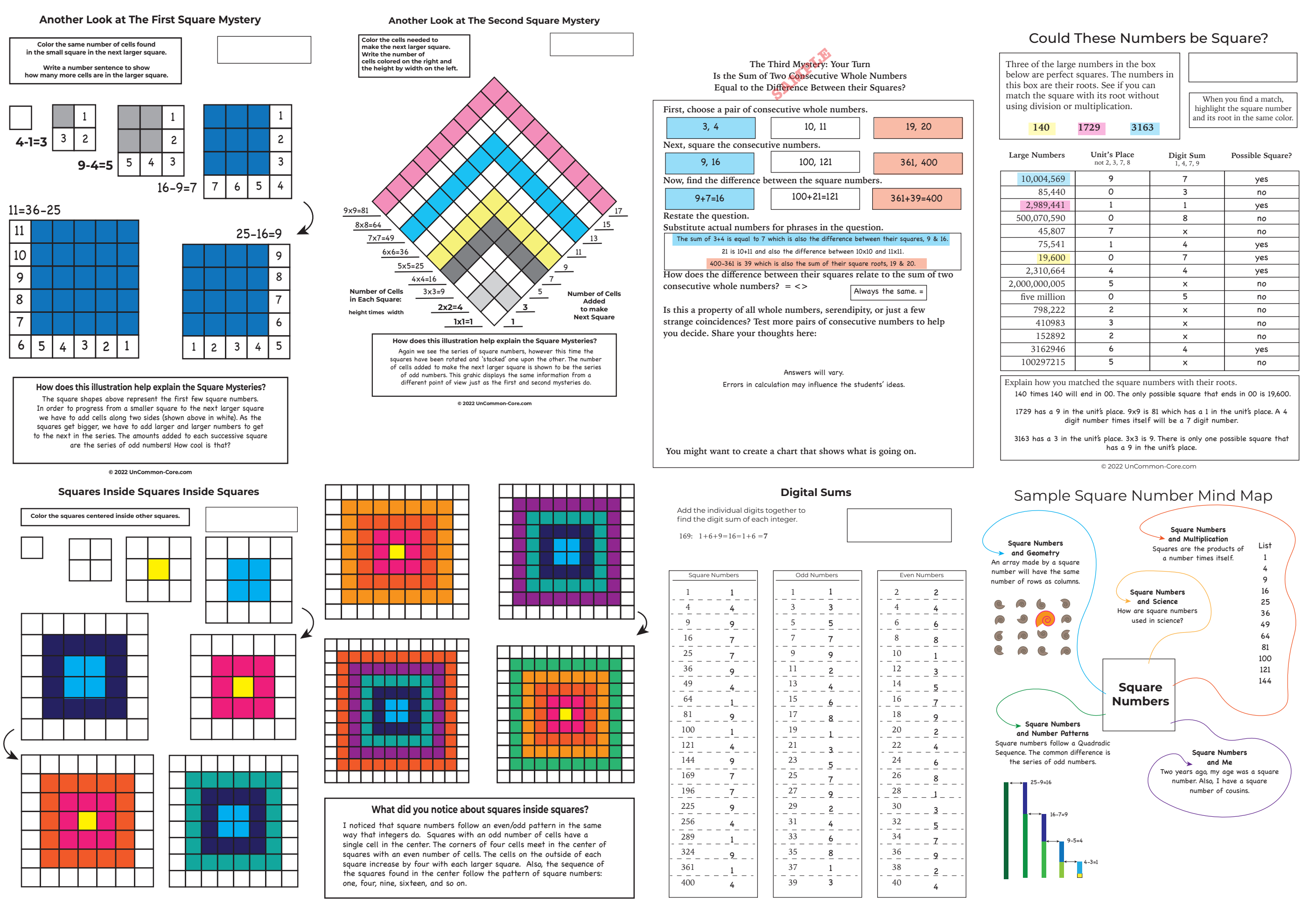
[NRICH Maths: square number activities]

[Number Sense Example: Ramanujan]

[Prime Number Theorem]

[Sum of Consecutive Odd Numbers]

[UnCommon-Core.com]



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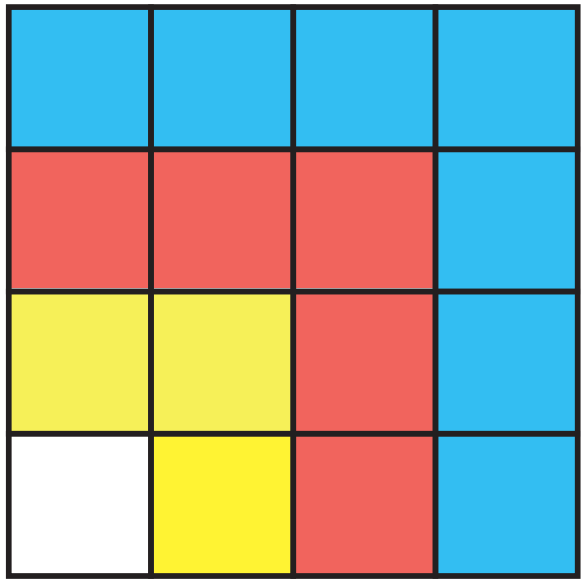

1 + 3 + 5 + 7

Does this work for every square?

We hope **Square Mysteries** will grab children by their imaginations and never let go. Students will become life long math lovers; always searching for interesting numerical relationships. Along with their spiralling interest, students' number sense and mathematical proficiency will soar to incredible heights.

Dream with us.