

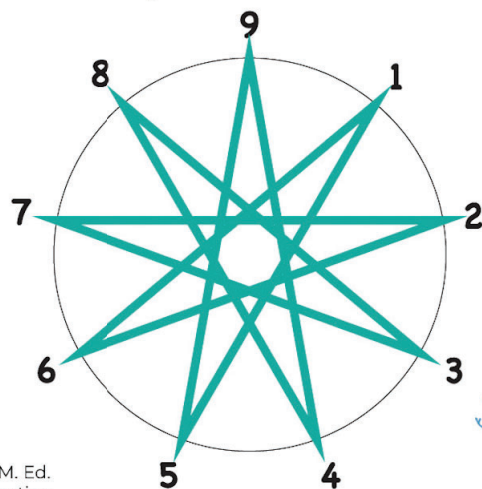
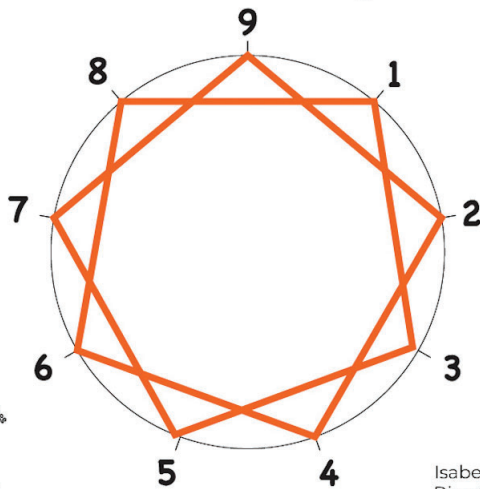
# Serious Math

# Serious Fun

## Patterns Found in Digit Sums of Multiples from Nine to One

### Serious Math ~ Serious Fun

#### Patterns Found in Digit Sums of Multiples from Nine to One



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grab students' imaginations

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Hello Teachers,

Thank you for downloading this handout. After decades of teaching, now I am sharing some of the activities I designed for my students and some new ones as well.

Please, check out the self-paced teacher education courses on UnCommon-Core.com.

While you are there, sign up for your free copy of **Colorful Collections: A Mindful Exploration of Proper Fractions**.

Also, visit my Teachers Pay Teachers store UnCommon-Core dot com.

Thank you again. All the best,



Isabelle

Isabelle Hoag M.Ed.  
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## Teacher Tips

What do you want your students to gain by exploring digit sums of multiples of numbers from one to nine?

Do elementary students need to:

- Practice addition? YES!
- Practice multiplication? YES!
- Make sense of directions? YES!
- Use math vocabulary correctly? YES!
- Display data in various formats? YES!
- Compare graphic displays of data? YES!
- Contrast graphic displays of data? YES!
- Understand place value? YES!
- Develop number sense? YES!
- Get excited about math? YES!
- Know about digit sums? (Not really, shhh!)

This handout supports the ideas introduced in the video of the same name on [the UnCommon-Core YouTube channel](#).

# Serious Math

# Serious Fun

## Patterns Found in Digit Sums of Multiples from Nine to One



# Basic Attributes

The name of each shape is [given in the video](#) when the shapes first appear. You might want to pause the video long enough for students to copy it onto their Attribute Charts.

A big question that runs through the video begins with: will all the shapes be the same or not, then becomes: will they always have the same design as the number they are paired with, and finally ends with asking how digit sums of multiples of nine can match the 'pattern' made by zero ~ when multiples of zero didn't make any pattern at all!

The shapes contain either a triangle or a nonagon. Smaller and smaller nonagons are visible inside the shapes made by two, seven, four, and five. Even though the shapes look different, they are made with either zero, three, or nine lines.

Likewise, the shapes have either zero, three, or nine points.

name: \_\_\_\_\_

Attribute Chart 1				
Attributes	Shape Name?	Same Design?	Number of Sides	Number of Points
Number Pairs				
0 + 9		As each other?		
		As other pairs?		
1 + 8		As each other?		
		As other pairs?		
2 + 7		As each other?		
		As other pairs?		
3 + 6		As each other?		
		As other pairs?		
4 + 5		As each other?		
		As other pairs?		

# Advanced Attributes

These attributes are in the order of difficulty rather than the order of presentation.

When the designs are being created the shape based on the smaller number is usually made clockwise and the shape based on the larger number is most often made counterclockwise.

Shapes that can be folded in half so that both sides match have mirror symmetry, also called reflectional symmetry. The axis of symmetry is the line dividing the shape into two equal parts. Triangles have three lines of symmetry. Regular nonagons have nine.

When a shape can be rotated around a central point and look the same at specific places along the way, the shape has rotational symmetry. Triangles have 3-fold rotational symmetry. With each rotation of  $60^\circ$  the triangle will look as if it hasn't been moved.

Chirality refers to having a left-hand and right-hand version. Shoes, gloves, mittens, and scissors are chiral. Shapes that look the same front and back or in the mirror are achiral.

name: \_\_\_\_\_

Attribute Chart 2				
Attributes Number Pairs	Direction of Creation	Mirror Symmetry	Rotational Symmetry	Chirality
0 + 9				
1 + 8				
2 + 7				
3 + 6				
4 + 5				

# Finding Digit Sums

Simply add up all the individual digits in the number. Continue until there is only one digit left. This is the **digit sum** of the number.

For example: the digit sum of 7,325 is eight:

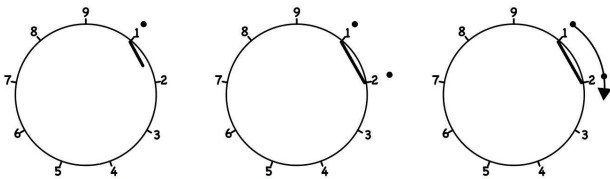
$$7 + 3 + 2 + 5 = 17 \quad 1 + 7 = 8$$

# Clockwise or Counterclockwise?

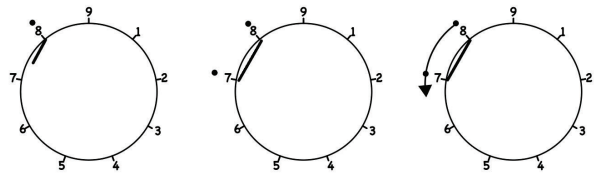
In which direction were the shapes created?

As you plot these designs around a circle, mark some points outside the circle that follow each new end point. Draw a line to connect the dots and add an arrow at the end. This arrow will show the direction in which the shape is made.

Growing in the direction that clock hands move:



Growing in the opposite direction from the clock hands:

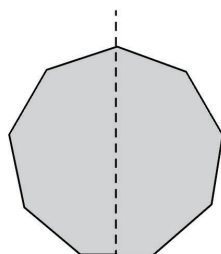


# Mirror Symmetry

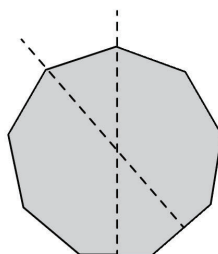
When a shape is folded in half, so that each side looks like the other, then it has **mirror symmetry**. Each fold down the center is called an **axis of symmetry**.

How many axes of symmetry does each shape have?

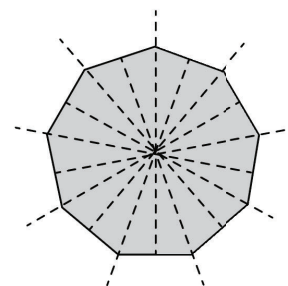
Cut out the shapes in order to test them. See how many different ways they can be folded into two equal halves. Count the lines of symmetry.



1 axis of symmetry



2 axes of symmetry

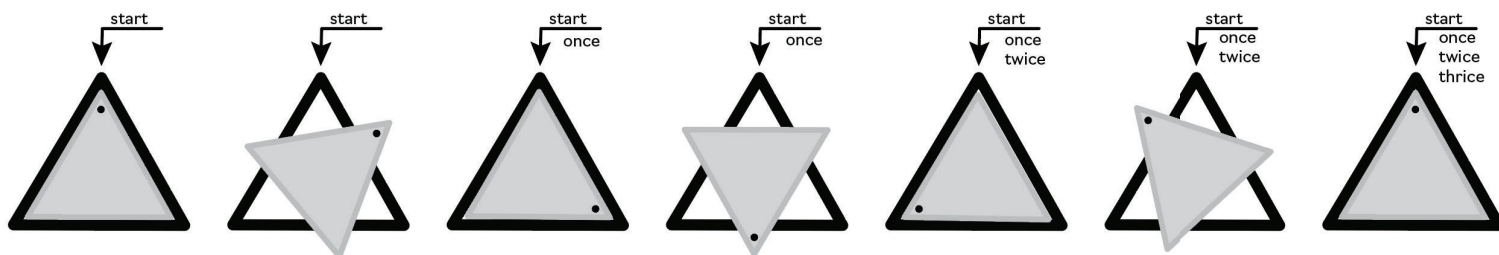


9 axes of symmetry

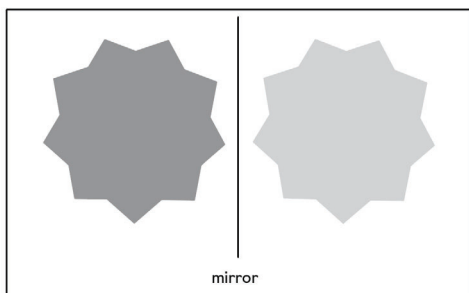


# Rotational Symmetry

Find out if a shape has **rotational symmetry** by tracing an outline of the shape. Then mark one corner of the shape and the outline to indicate a starting point. Place your finger ~ or a push pin ~ in the center of the shape and gently turn it around until it fits inside the outline again. Keep track of how many times the shape fits into the outline before ending up back where it started.

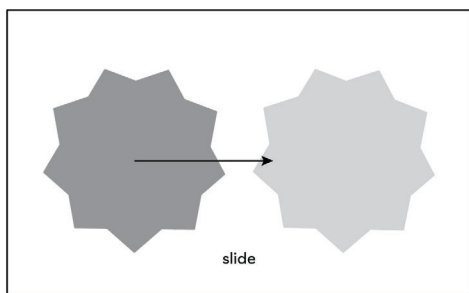


## Chiral or Achiral?

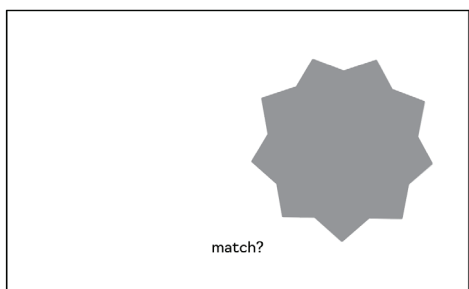


**Achiral** means the image and its reflection can be placed on top of each other and still match.

Hold your hands palms up in front of you. Slide them together so that one is on top of the other. Do they match? Nope. **Chiral** means there is a left hand shape and a right hand shape.



Cut out two of each shape (or work with a friend). One will be the reflection. Flip one of the shapes upside down. Carefully slide one shape on top of the other. You may rotate the top shape clockwise or counterclockwise to see if they will match. If they match perfectly, then the shape is achiral. If the shape doesn't match its reflection then it is chiral.

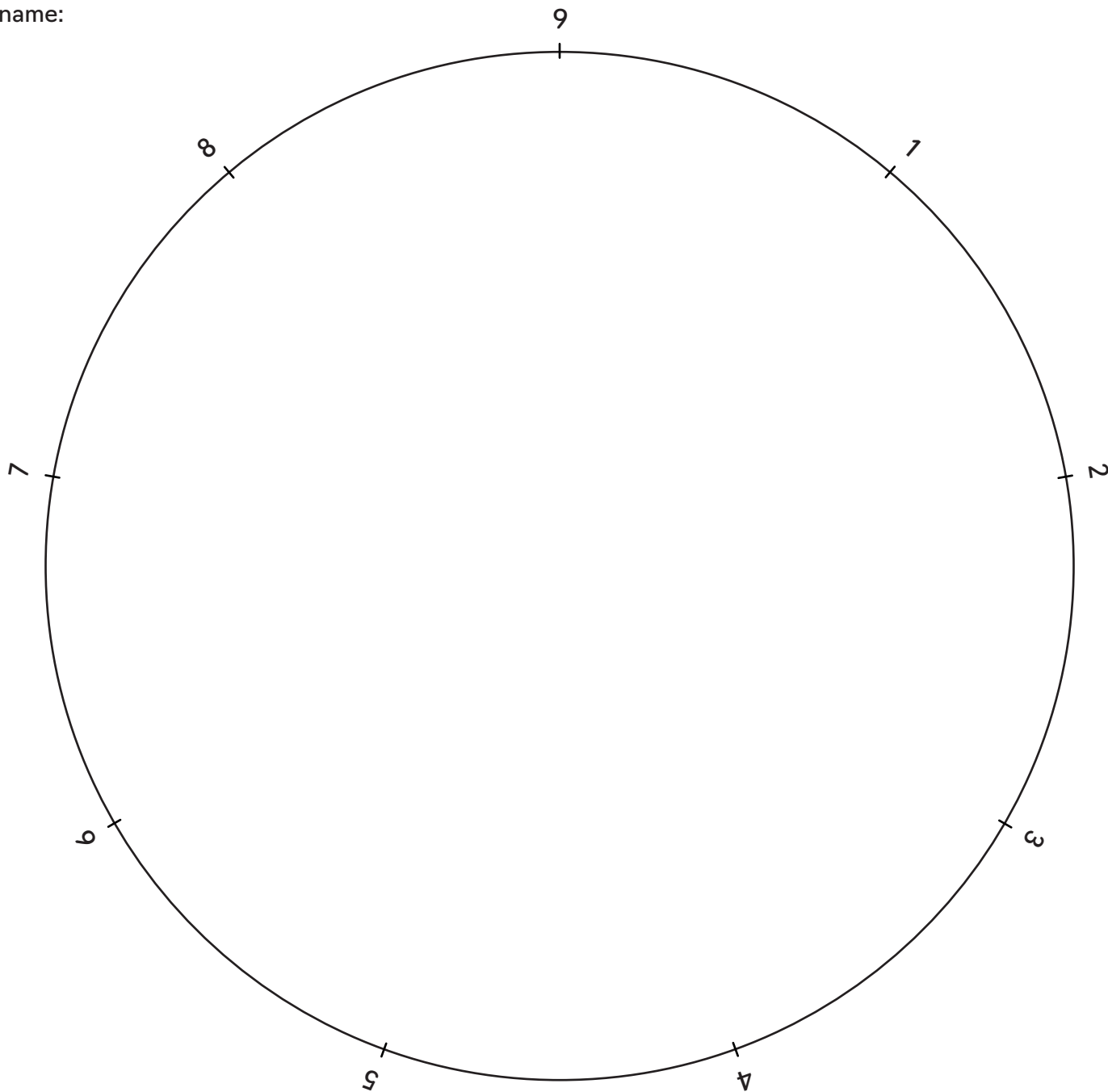


Think about the capital letters. Which ones could be flipped over or reflected in a mirror and still match?

ABCDEFGHIJKLMNOPQRSTUVWXYZ

# Digit Sum Designs in Multiples of:

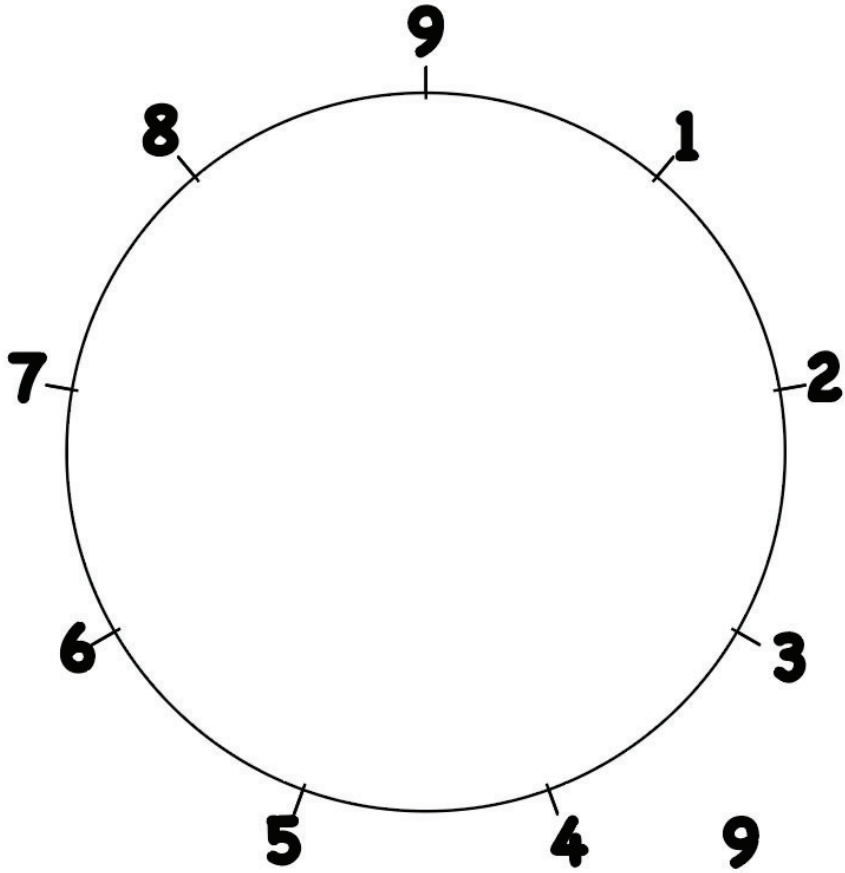
name: \_\_\_\_\_



Digit Sums: \_\_\_\_\_

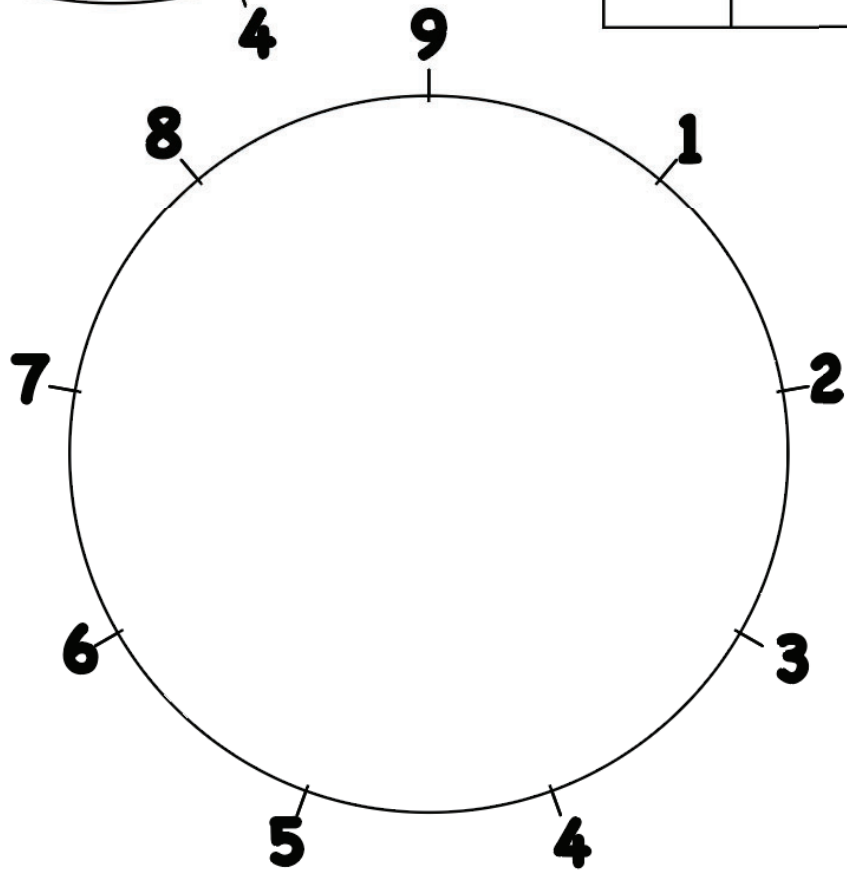
Multiples of: \_\_\_\_\_

Choose a number between one and nine. Write the first ten multiples of your number in order on the first line. Then write their digit sums on the line above. Find the digit sum by adding the numerals in the number together. Keep adding until you have a single digit left. That digit is the digit sum of the original number. Next, use a ruler to help connect the digit sums in order. What designs will digit sums of multiples of different numbers make?












x	DS

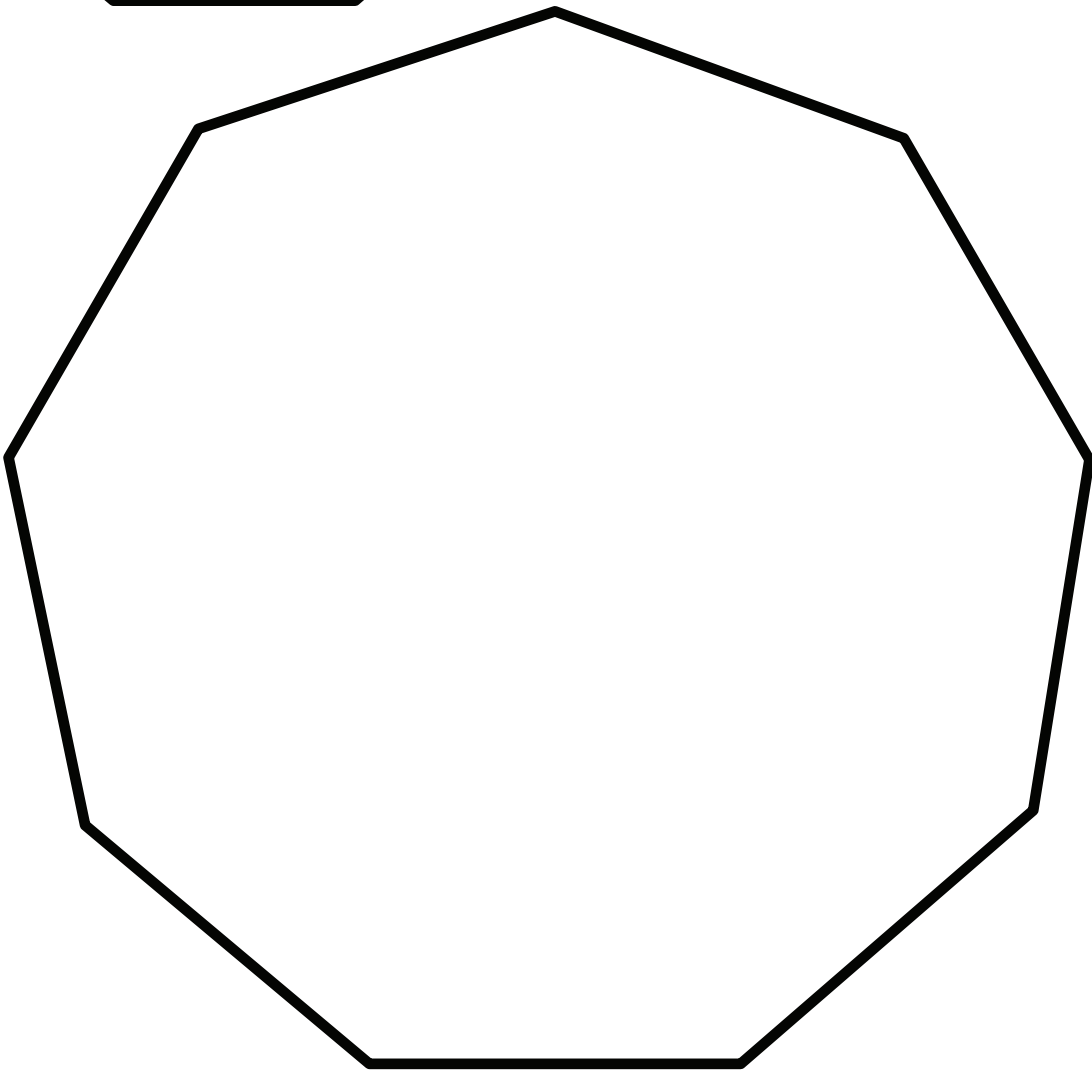
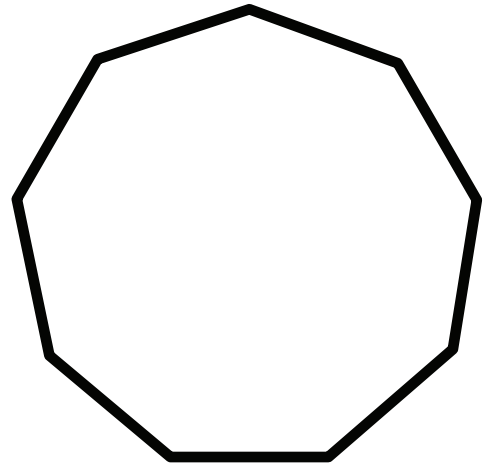
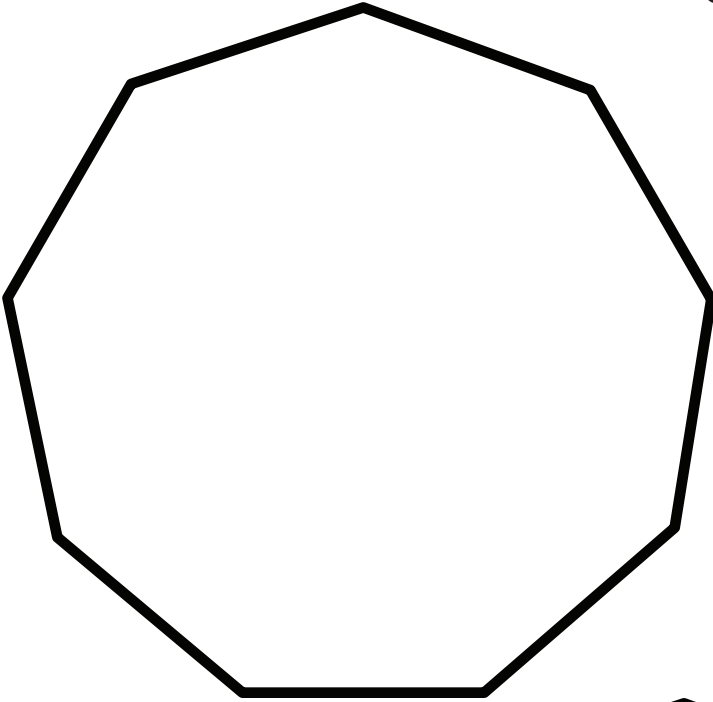
x	DS



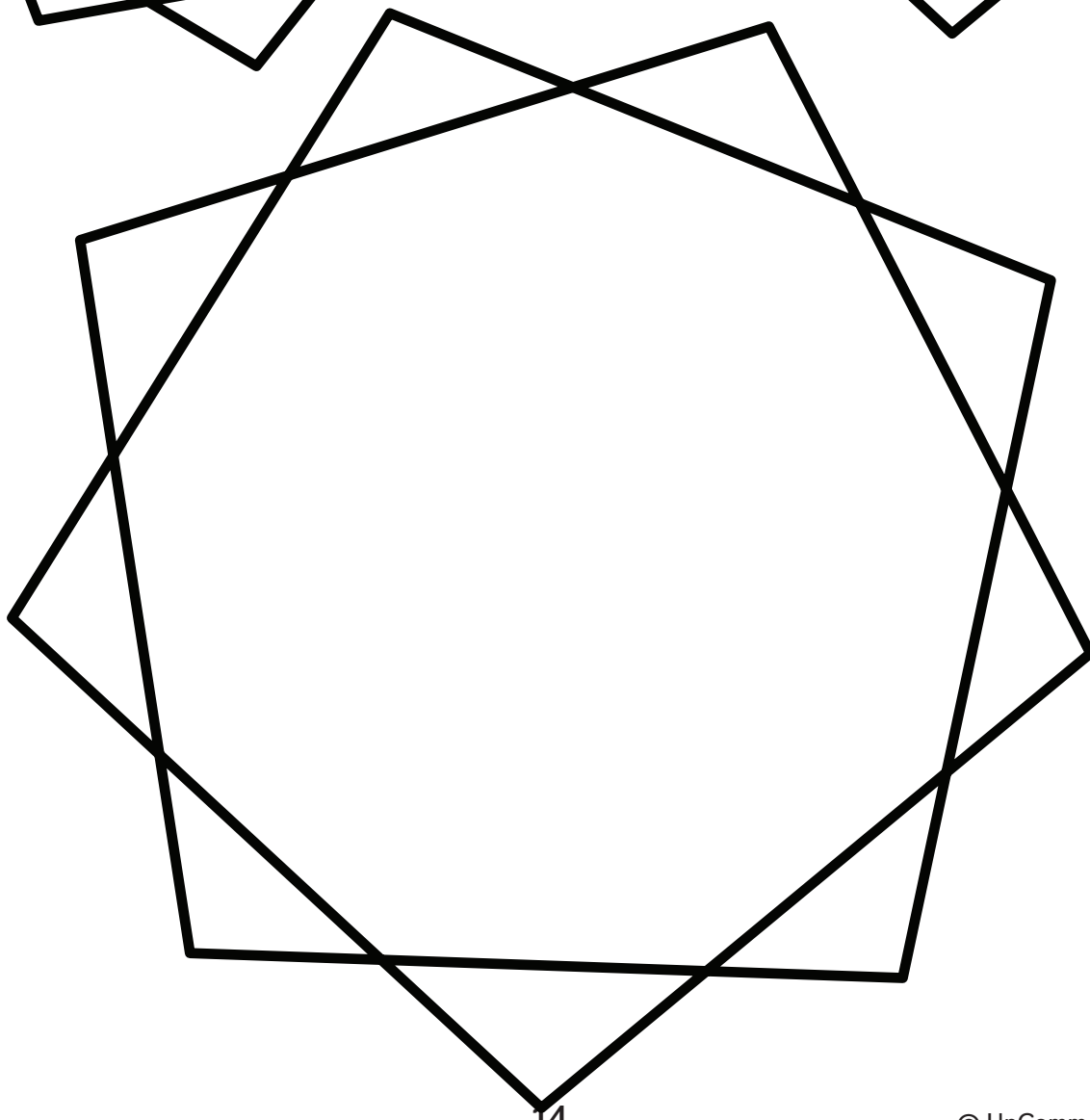
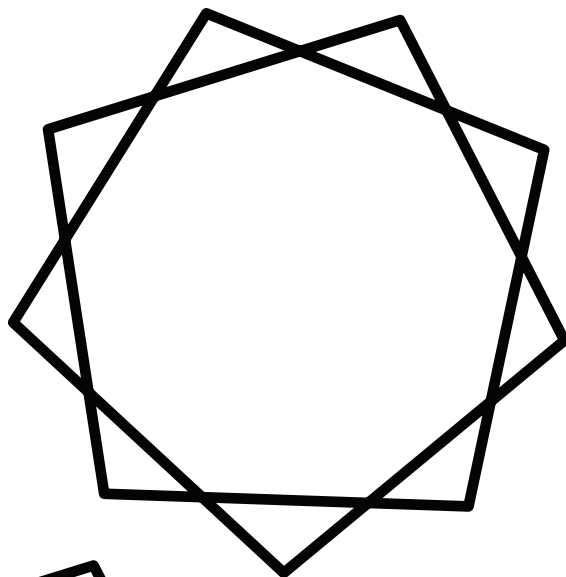
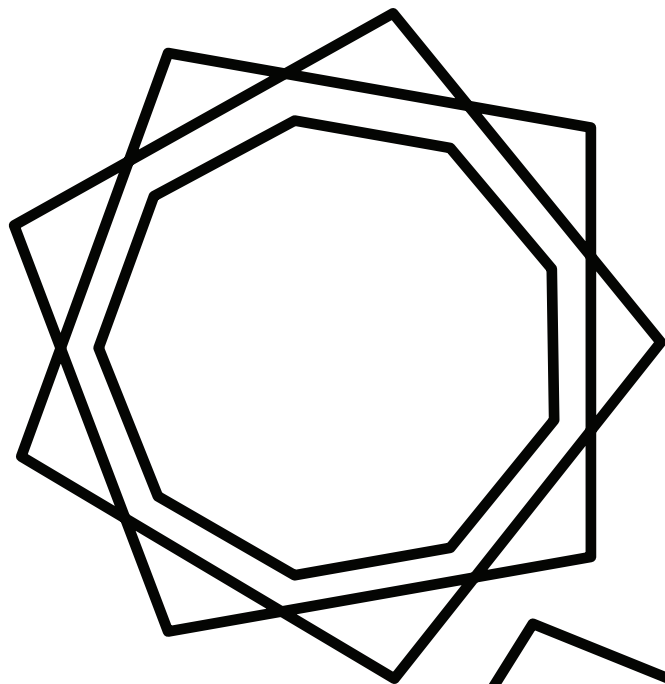
What kinds of designs will digit sums of multiples of two digit numbers make?

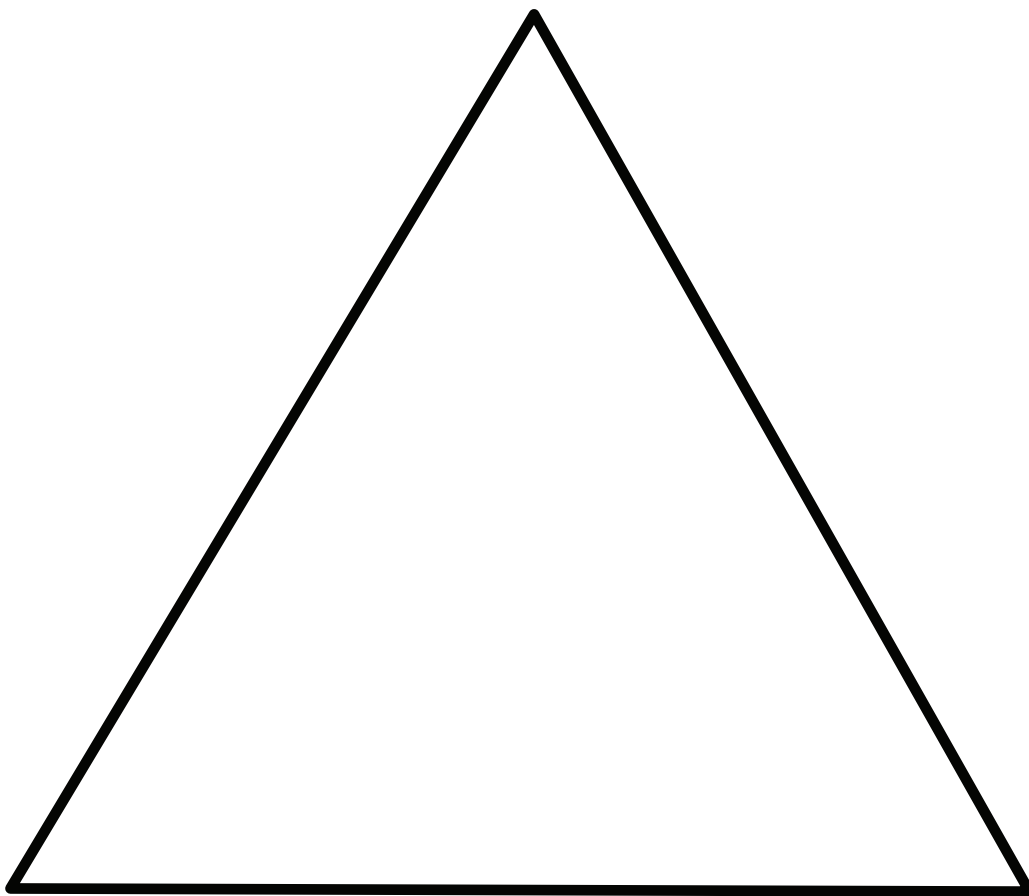
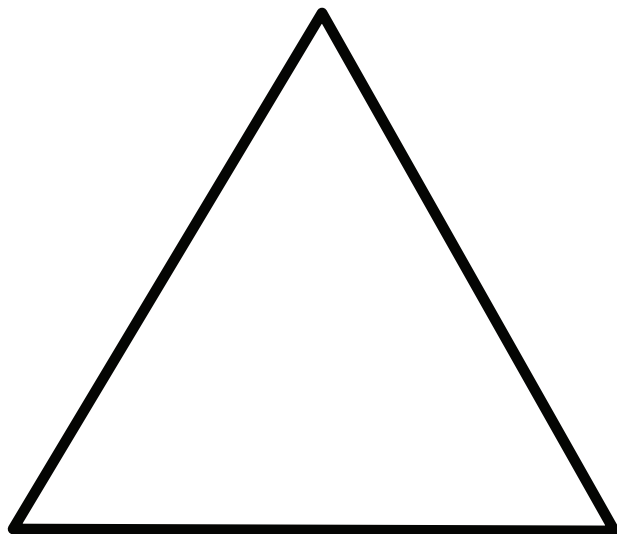
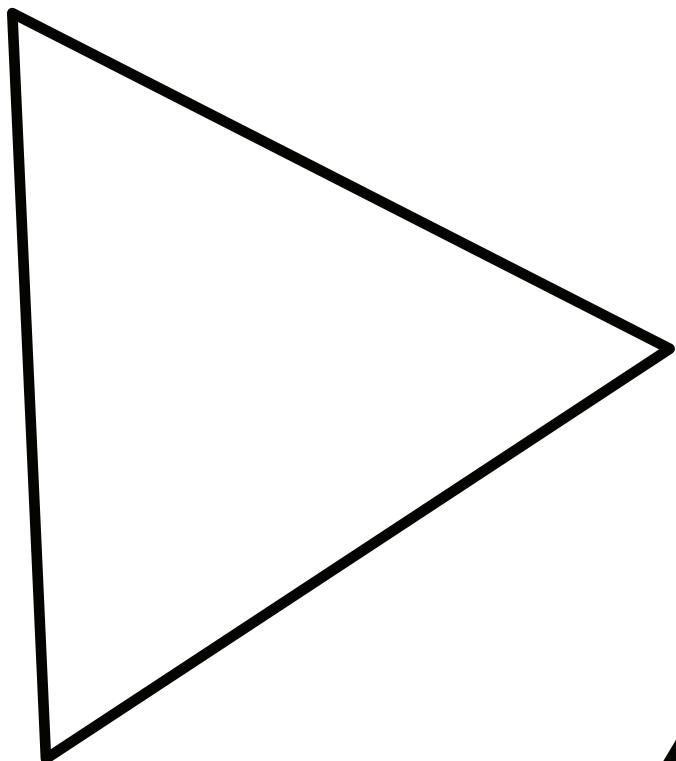
# Nonagon



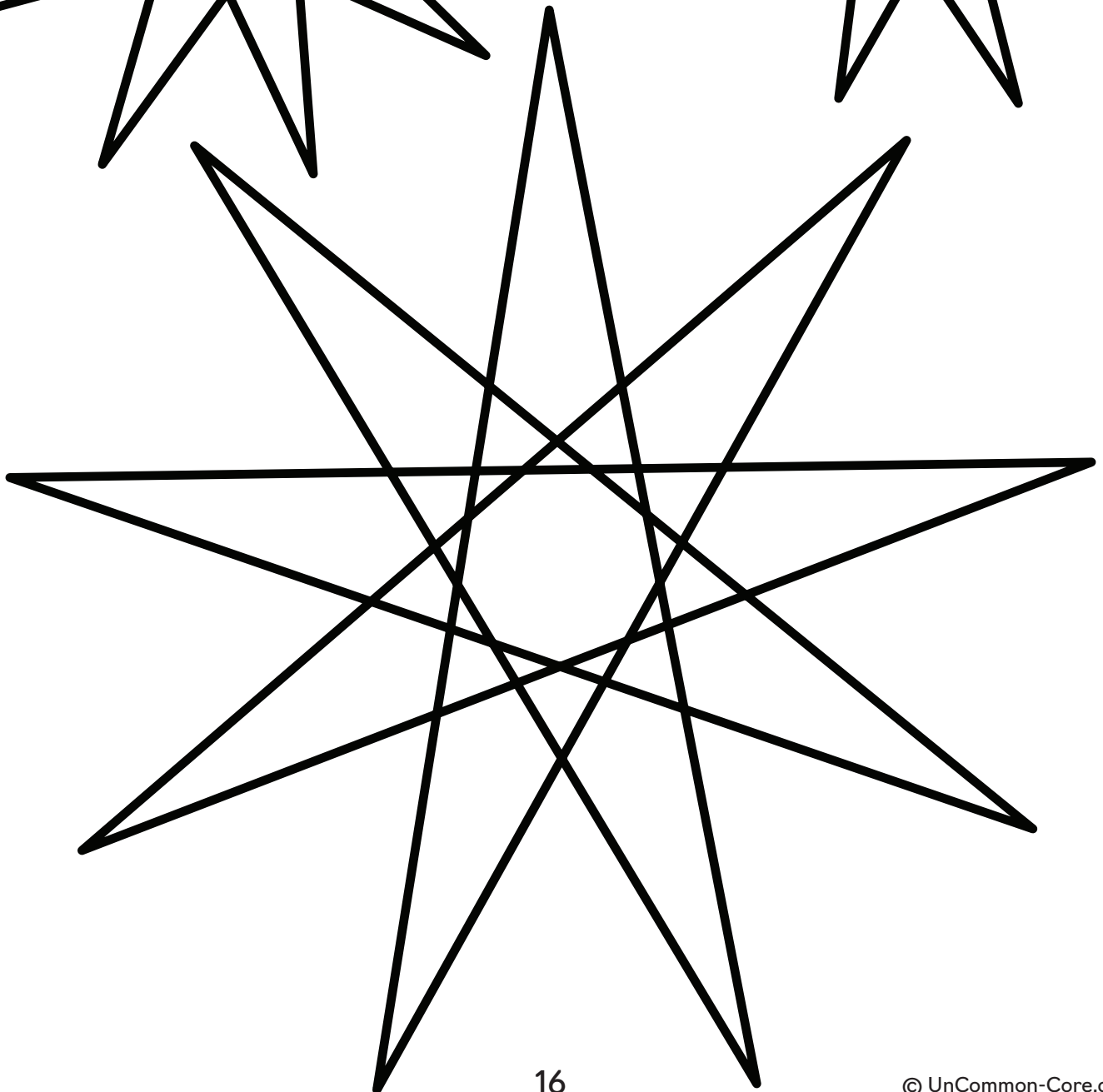
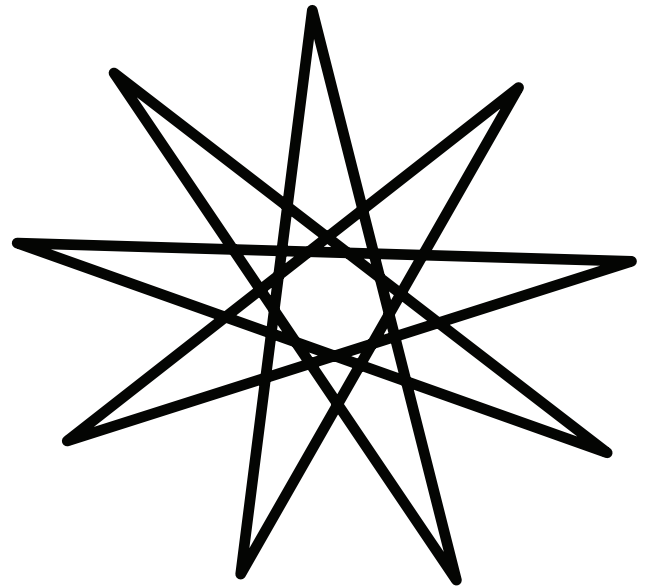
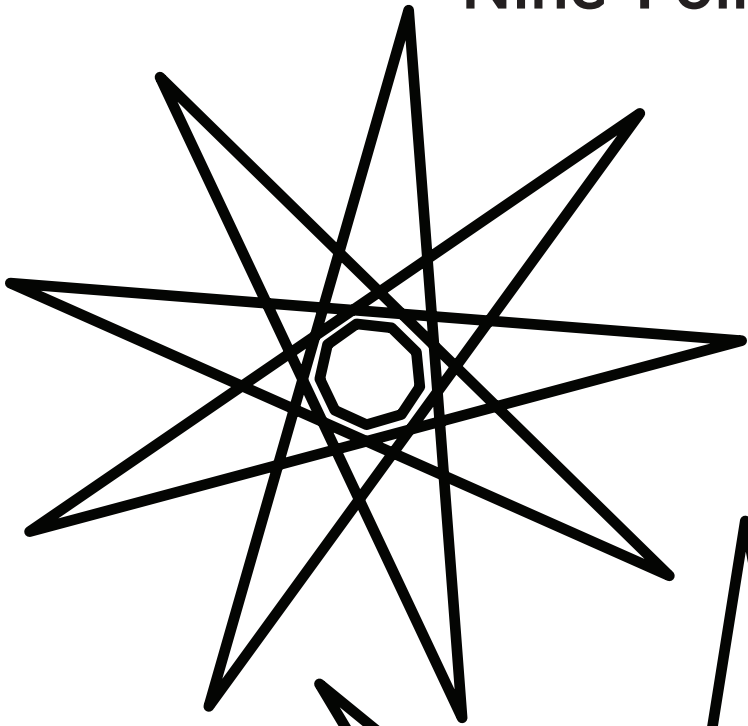
# Nine-Pointed Sun



# Triangle



# Nine-Pointed Star

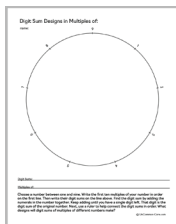




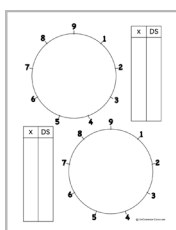
Everything needed to [‘play math’ with the video](#) is provided in this handout.

Have students decorate and write their names on the cover of the activity folder if using.

Either, let them make their own shapes, or copy those below. Students will use the shapes to test for various attributes. Encourage students to write on the shapes in order to track their path as they count sides and points, rotations or reflections. Keep instructions, pages, shapes, and work in progress in an activity folder as needed.



Have plenty of copies of the two pages on the left. The page on the top is the same one used at the beginning of the video to create the triangle made by the digit sums of multiples of three.



The page under it has smaller circles, and vertical spaces for the multiples and the digit sums. This form lets students make two designs at the same time. They can use this form to show the designs made by number pairs that add to nine on the same page.

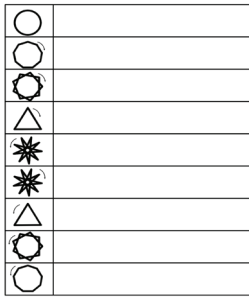
Attribute Chart 1				
Attributes	Shape Name?	Same Design?	Number of Sides	Number of Points
Number Pairs				
0 + 9	N/A	As each other? yes	N/A	N/A
		As other pairs? N/A		
1 + 8	nonagon	As each other? yes	nine	nine
		As other pairs? no		
2 + 7	nine-pointed sun	As each other? yes	nine	nine
		As other pairs? no		
3 + 6	triangle	As each other? yes	three	three
		As other pairs? no		
4 + 5	nine-pointed star	As each other? yes	nine	nine
		As other pairs? no		

## Think about capital letters. Which ones could be flipped over and still match?

Well, first, it depends on the font. Next, it depends on where you put the mirror.

Achiral when flipped left to right: A H I M O T U V W X Y

Achiral when flipped top to bottom (you can rotate them to match): A B E H I M O T U V W X Y



Grab a calculator and let your students find the first few digit sums of multiples of two digit numbers. They will be amazed to see which numbers are at the heart of each design.

What kinds of designs will digit sums of multiples of two digit numbers make?

	0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
	1, 10, 19, 28, 37, 46, 55, 64, 73, 82, 91
	2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92
	3, 12, 21, 30, 39, 48, 57, 66, 75, 84, 93
	4, 13, 22, 31, 40, 49, 58, 67, 76, 85, 94
	5, 14, 23, 32, 41, 50, 59, 68, 77, 86, 95
	6, 15, 24, 33, 42, 51, 60, 69, 78, 87, 96
	7, 16, 25, 34, 43, 52, 61, 70, 79, 88, 97
	8, 17, 26, 35, 44, 53, 62, 71, 80, 89, 98

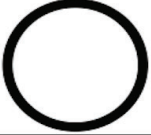


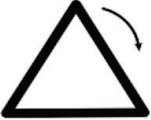





If that isn't enough excitement, you could make copies of the answer key (left) and let your students find the digit sums of every number on the page.

Hint: Numbers in each row will have the same digit sums. This is why their multiples create the same patterns.

## Attribute Chart 2

Attributes	Direction of Creation	Mirror Symmetry	Rotational Symmetry	Chirality
Number Pairs				
0 + 9	N/A	N/A	N/A	N/A
1 + 8	1 clockwise 8 counterclockwise	yes 9 fold	yes 9 fold	achiral
2 + 7	2 clockwise 7 counterclockwise	yes 9 fold	yes 9 fold	achiral
3 + 6	3 clockwise 6 counterclockwise	yes 3 fold	yes 3 fold	achiral
4 + 5	5 clockwise 4 counterclockwise	yes 9 fold	yes 9 fold	achiral

What kinds of designs will digit sums of multiples of two digit numbers make?

	0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
	1, 10, 19, 28, 37, 46, 55, 64, 73, 82, 91
	2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92
	3, 12, 21, 30, 39, 48, 57, 66, 75, 84, 93
	4, 13, 22, 31, 40, 49, 58, 67, 76, 85, 94
	5, 14, 23, 32, 41, 50, 59, 68, 77, 86, 95
	6, 15, 24, 33, 42, 51, 60, 69, 78, 87, 96
	7, 16, 25, 34, 43, 52, 61, 70, 79, 88, 97
	8, 17, 26, 35, 44, 53, 62, 71, 80, 89, 98

# Answer Key

X1	DS
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	1

X8	DS
8	8
16	7
24	6
32	5
40	4
48	3
56	2
64	1
72	9
80	8

X2	DS
2	2
4	4
6	6
8	8
10	1
12	3
14	5
16	7
18	9
20	2

X7	DS
7	7
14	5
21	3
28	1
35	8
42	6
49	4
56	2
63	9
70	7

X3	DS
3	3
6	6
9	9
12	3
15	6
18	9
21	3
24	6
27	9
30	3

X6	DS
6	6
12	3
18	9
24	6
30	3
36	9
42	6
48	3
54	9
60	6

X4	DS
4	4
8	8
12	3
16	7
20	2
24	6
28	1
32	5
36	9
40	4

X5	DS
5	5
10	1
15	6
20	2
25	7
30	3
35	8
40	4
45	9
50	5

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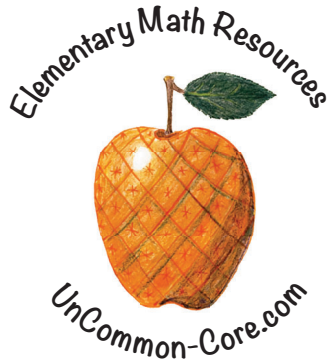
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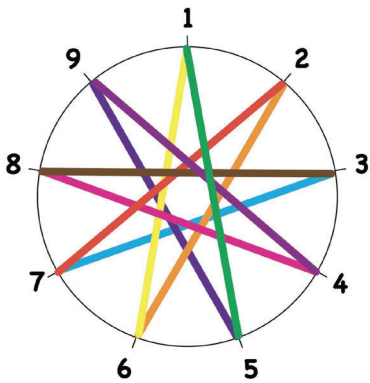
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[Isabelle@UnCommon-Core.com](mailto:Isabelle@UnCommon-Core.com)

# Try These NEXT!

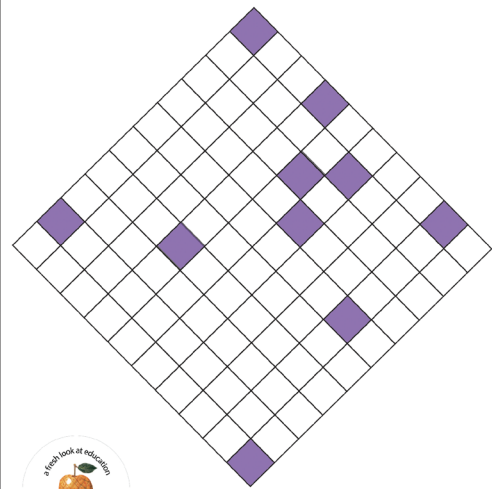


## Explore Digit Sums Practice Math Skills



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## Square Mysteries



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How clearly can your students  
see what you are teaching them?

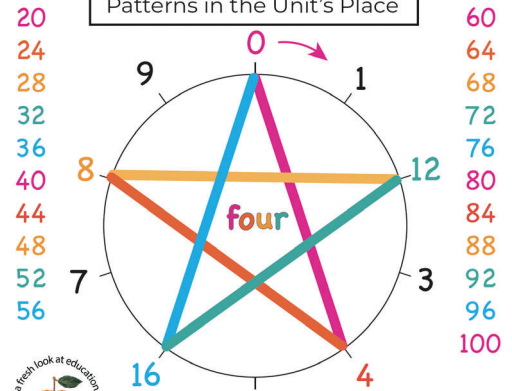
Is their understanding rosy? Smudged? Out of focus?

Check in with your students  
using this lighthearted reflection tool:

## How Are Your Glasses?



## Multiplication Math Art Patterns in the Unit's Place



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